During the construction and occupancy of heavy structures (nuclear power plants, hydroelectric power plants, etc.), which are built on a compressible bed, the need arises to predict their settlements and tilting [1, 2].

The solution of this problem requires a large volume of long-term experimental studies and calculation of the stress-strain state (SSS) of the bed. Successful solution of this problem is governed by both the correct selection of the geomechanical model of the bed, and the accuracy with which the rheologic properties of the bed soils and the effectiveness of the adopted computational method are described.

Studies conducted in recent years have shown that a rheologic model of the bed in the form of a layer of limited thickness is most optimal [3-7].

Investigation of clayey soils suggests that a rheologic model where the deformations are logarithmically proportional with time is effective for predicting long-term deformations. A theoretical solution of the problem of predicting the SSS of a soil layer, which is inhomogeneous with respect to depth, under a local load with allowance for the rheologic properties of the soils is proposed below on the basis of these two premises, using Vlasov’s variational method [7], which ensures sufficient accuracy and practicality.

Let us assume that nonuniformly elastically creeping beds are represented as a compressible layer of thickness $H$, which are situated on an infinitely rigid half space (Fig. 1).

Studies performed by Ter-Martirosyan [8] and Meschayan [9] make it possible to state that the behavior of single-phase soils conforms to the linear theory of hereditary creep and satisfy the requirements of the basic premises of the Maslow-Arutyunyan theory of an elastically creeping body [10], and that the coefficient of transverse creep strain $\eta(t)$ is equal to the coefficient of transverse elastic strain $\eta(t)$, and is constant with time

$$v_2(t, \tau, \gamma) = v_1(t, \gamma) = v = \text{const.}$$

In this case, the equations of state of a nonuniformly elastically creeping bed are defined by the relationships

$$
\begin{align*}
\sigma_1(t) &= \left[\frac{N(t, y)}{1 - v_1(t)}[\sigma_1(t) - v_2(t)]\right]; \\
\sigma_2(t) &= \left[\frac{N(t, y)}{1 - v_2(t)}[\sigma_2(t)]\right]; \\
\sigma_3(t) &= \left[\frac{N(t, y)}{2(1 + v_1(t))[\sigma_1(t) + v_2(t)]}\right]; \\
\tau_{xy}(t) &= \left[\frac{N(t, y)}{2(1 + v_1(t))[\sigma_1(t) + v_2(t)]}\right];
\end{align*}
$$

(2')

where

$$
\begin{align*}
M(t, y) &= \frac{1}{E(t, y)}[1 - K^*]; \\
N(t, y) &= E(t, y)(1 + K^*); \\
K^* &= \int K(t, \tau, y) / (\tau - t) d\tau; \\
R^* &= \int R(t, \tau, y) E(t, y) / (\tau - t) d\tau;
\end{align*}
$$

(3)

(4)

\[ R(t, r, y) = \text{resolvent nucleus } K(t, r, y); \]
\[ K(t, r, y) = E(t, y) \left( \frac{\partial}{\partial t} \right) \frac{1}{1/E(t, y) + C(t, r, y)}; \]
\[ E(t, y) = E_0(t, y) / \left[ 1 - \phi_0(y) \right]; \]

and \( E_0(t, y), C(t, r, y) \) and \( \phi_0(y) \) are, respectively, the compression modulus, a creep measure, and Poisson’s ratio of the nonuniformly elastically creeping soil.

According to the principle of possible displacements, the equilibrium conditions of an elementary strip of width \( dx = 1 \) and height \( H \), which is isolated from the bed layer, can be represented as (see Fig. 1)

\[
\begin{align*}
\int [\rho_{\nu_2}(t, r) \frac{\partial}{\partial x} \nu_2(r, y)] dx - \int \tau_{\nu_2}(t, r) \frac{\partial}{\partial y} \nu_2(r, y) + \int \rho(t, r, y) dy = 0, \\
\int \left( \frac{\partial}{\partial x} v_k(t, r) \right) dy - \int \tau_{\nu_2}(t, r) v_k(t, r) + \int q_k(t, r, y) dy = 0,
\end{align*}
\]

(5)

where \( \nu_2(r, y) \) and \( v_k(t, r) \) are unknown dimensionless functions characterizing the transverse distribution of bed displacements.

Introducing expressions (2') in equilibrium equation (7) and using expansion (8), we obtain \( m + n \) integrodifferential equations in terms of the unknown functions \( U_i(t, x) \) and \( V_k(t, x) \):

\[
\begin{align*}
\sum_{i=1}^{m} a_{ij} \frac{d^2 U_i}{dt^2} + 2(1 - \nu_0) \sum_{i=1}^{m} a_{ij} \frac{d U_i}{dt} + \sum_{k=1}^{n} k_{ij} \frac{d}{dt} U_i + \frac{1}{2} \left[ \sum_{k=1}^{n} \frac{\phi_k(t)}{E_{ok}(1)} \left( \frac{\partial}{\partial x} \sum_{k=1}^{n} \phi_k(t) \frac{\partial}{\partial y} U_i \right) \right] + \sum_{k=1}^{n} \sum_{l=1}^{n} a_{lk} V_l = 0, \\
\sum_{i=1}^{m} \phi_i(t) \frac{\partial}{\partial x} U_i + \frac{1}{2} \left[ \sum_{k=1}^{n} \frac{\phi_k(t)}{E_{ok}(1)} \left( \frac{\partial}{\partial x} \sum_{k=1}^{n} \phi_k(t) \frac{\partial}{\partial y} U_i \right) \right] + \sum_{k=1}^{n} \sum_{l=1}^{n} a_{lk} V_l = 0, \\
\end{align*}
\]

(9)

where \( a_{ij}, b_{ij}, \ldots, c_{hk}, \tilde{s}_{hk} \) are integral operators.

The generalized longitudinal and transverse forces in the section \( x = \text{const} \) are represented as:

\[
\begin{align*}
l_j(t) &= \left[ E_{ok}(1) / (1 - \nu_0) \right] \sum_{i=1}^{m} s_{ij} \frac{d U_i}{dt} + \nu_0 \sum_{k=1}^{n} \frac{\phi_k(t)}{E_{ok}(1)} \frac{\partial}{\partial x} U_i, \\
S_k(t) &= \left[ E_{ok}(1) / (1 + \nu_0) \right] \sum_{i=1}^{m} \phi_i(t) \frac{\partial}{\partial x} U_i + \sum_{k=1}^{n} \frac{\phi_k(t)}{E_{ok}(1)} \frac{\partial}{\partial y} U_i.
\end{align*}
\]

(10)

System of equations (9) describes the generalized model of a nonuniformly elastically creeping bed of finite thickness, which is constructed on the basis of the general variational method outlined by Vlasov and Leont’ev [7].

To illustrate the use of this method, let us determine the settlement and SSS of the beds of heavy structures.

Let us assume that the lateral displacements in the layer are very small and can be neglected, and there are no vertical displacements along the lower surface of this layer. The displacements in this layer, which are induced by a surficial load, can then be approximately represented as.