EFFECT OF GROUP HETEROGENEITY ON
ITEM PARAMETERS*

HAROLD GULLIKSEN
EDUCATIONAL TESTING SERVICE

Most indexes of item validity and difficulty vary systematically with changes in the mean and variance of the group. Formulas are presented showing how certain item parameters will vary with these alterations in group mean and variance. Item parameters are also suggested which should remain invariant under such changes. These parameters are developed under two different assumptions: first, the assumption that the total distribution of the item ability variable is normal, and, second, that the distribution of the item ability variable for each array of the explicit selection variable is normal.

Most indexes of item validity or item difficulty vary systematically with the ability level of the group and with the variance in the ability of the group. Thus, the "per cent of persons answering an item correctly" increases as the mean ability of the group increases and decreases as the mean ability of the group decreases. The "item-criterion correlation" increases and decreases as the variance of the group increases and decreases. Item parameters which did not vary systematically as the mean and variance of the group ability changed would be valuable. Lacking such parameters it would be valuable to have formulas indicating the amount of change in a given item parameter to be expected for a given change in group mean and variance.

This paper will present the derivation of formulas which show the amount of change in various item parameters which should occur because of changes in group heterogeneity and also will develop item parameters which do not systematically increase or decrease as the heterogeneity of the group changes.

We will consider two groups of persons. An unselected group with a specified mean and variance and a selected group with a different mean (usually though not necessarily higher) and a different variance (usually though not necessarily smaller). It should also be noted that the theory to be developed applies regardless of the direction of the change in mean and variance. Thus, rather than utilizing

*The writer wishes to acknowledge helpful discussions of this paper with Paul Horst and Herbert S. Sichel who have worked on various aspects of the problem of invariant item parameters.
one set of symbols to designate the selected group and another set to
designate the unselected group we will say that lower-case letters will
be used to designate the group for which all the parameters are avail-
able; upper-case letters will be used to designate the group for which
some information is available and for which information is desired on other parameters. That is to say, the unknowns to be solved for,
will always be designated by upper-case letters. It will be noted dur-
ing the derivation that no assumption is made regarding the direction
of the change so that the formulas hold for utilizing information
from an unselected group to estimate parameters in a selected group
or for utilizing parameters from a selected group to estimate those
in an unselected group.

I. Definitions and Assumptions

We will consider three major types of variables, as follows:

\( X, x \) is used to designate the explicit selection variable,
that is, the variable which was directly used for se-
lection.

\( Y, y \) is used to represent any other variable on which se-
lection occurs only because of its correlation with
\( X (x) \); this variable may be termed the variable sub-
ject to incidental selection.

\( I_o, i_o \) designates a gross score variable which represents
the ability required to answer item \( G (g) \).
\( (G = 1 \ldots K; g = 1 \ldots k) \). For example, for a test
of \( K (k) \) items there would be \( K (k) \) such variables,
a different one for each item.

It is also necessary to assume that for each item ability variable
there exists an ability level such that all persons above this level an-
swer the item correctly and all persons below this level answer the
item incorrectly.

\( I'_o, i'_o \) will be used to designate the particular ability score
above which all persons answer item \( G (g) \) correct-
ly and below which all persons answer item \( G (g) \)
incorrectly.

For any given item it is assumed that

\[ I'_o = i'_o \quad (G = g). \tag{1} \]