Remark 2. It is shown in [9] that for each Radon measure $\mu$ on $X$ quasi-invariant in all directions from $E$ and for each Borel mapping $F : X \to E$ Lipschitz along $E$ the Gateaux derivative along $E$ exists $\mu$-almost everywhere. On one hand, this means that the original mapping in the theorem above possesses the differentiability property, and on the other hand, this leads to the natural question of the extendability of this theorem to infinite-dimensional mappings.

Remark 3. The following close (but weaker) statement was obtained in the recent paper [10]: if $f$ is a function measurable with respect to a Gaussian measure $\mu$ on a separable Banach space $X$ defined on a $\mu$-measurable set $A$ and satisfying there the Lipschitz condition along the Cameron–Martin space $E$, then there exists a $\mu$-measurable function on $X$ satisfying the same condition and coinciding with $f$ $\mu$-almost everywhere. This statement follows from the theorem above, since $f$ has a Borel modification satisfying the same condition on a Borel subset $B \subset A$ with $\mu(B) = \mu(A)$. Our theorem implies that the same is true if $X$ is a locally convex space with the Radon Gaussian measure $\mu$ (in this case, according to [11], the measure $\mu$ is supported by a Suslin linear subspace).

References


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Phase Transition for Classical Bosons, Fermions, and Ordinary Classical Particles

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In the preceding paper, for classical fermions we considered the case of an attracting interaction potential $V(x_i - x_j)$ attaining its minimum at the point $x_i = x_j$. However, actual interaction potentials $V(z) \geq -A$, $A > 0$, are repulsive if the particles are at the same point and attain their minimum at some point $x \neq 0$. 

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One can show that in this case the self-consistent field equations for classical bosons, fermions, and particles obeying other parastatistics, as well as for classical particles, under a zero external field have the form

\[
\frac{\partial \rho_n}{\partial t} + \frac{p}{m} \frac{\partial \rho_n}{\partial x} - \frac{\partial W_n}{\partial x} \frac{\partial \rho_n}{\partial p} = 0, \quad x \in \mathbb{R}^3, \quad p \in \mathbb{R}^3,
\]

\[
W_n(x) = \int \int dy dp V(x - y) \left( \sum_{k \neq n} \rho_k(y, p) \right),
\]

where \( N \) is the number of particles, under the condition

\[
\sum_{k=1}^{N} \int \int \rho_k(y, p) dy dp = N.
\]

The averaged "dressed" potential \( W(x) \) is equal to

\[
W(x) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} W_n(x).
\]

We set

\[
\rho_n^\mp = \left( \exp \left\{ \frac{p^2/(2m) + W_n(x) - \mu}{kT} \right\} \pm 1 \right)^{-1}, \tag{3}
\]

respectively, for classical bosons and fermions and

\[
\rho_n = \exp \left\{ \frac{-p^2/(2m) - W_n(x)}{kT} \right\} \tag{4}
\]

for ordinary classical particles. Here the sign "\( - \)" corresponds to classical bosons and the sign "\( + \)" to fermions, \( m \) is the mass of a particle, \( \mu \) is the chemical potential, \( k \) is the Boltzmann constant, \( T \) is the temperature, and \( \partial \rho_n / \partial t = 0 \).

In particular, for \( T = 0 \) we have

\[
\rho_n^- = \delta \left( \mu - \frac{p^2}{2m} + W_n(x) \right), \quad \rho_n^+ = \theta \left( \mu - \frac{p^2}{2m} - W_n(x) \right).
\]

Consequently, for the classical bosons we have

\[
W_n(x) = \sum_{k \neq n} V(x - y_k), \tag{6}
\]

where the \( y_k \) are the points where \( \min_x W_k(x) = W_k(y_k) \) is attained.

Thus, we obtain the system of implicit relations

\[
\sum_{k \neq n} V(y_n - y_k) = \min_x \sum_{k \neq n} V(x - y_k). \tag{7}
\]

Relation (2) gives the self-consistent periodic potential corresponding to the original many-particle problem provided that \( W''(x_0) > 0 \), where \( x_0 \) is the minimum of \( W(x) \) corresponding to a crystal lattice. Hence at low temperatures there exist \( N \) distinct solutions of problem (1). The temperature at which the distinct solutions cease to exist and only the solution \( \rho_k = \rho_i \neq \text{const} \) for all \( k \) and \( i \) remains valid, that is, the