An Experimental Study of the Buckling of Open Cylindrical Shells

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Experimental determination of the load-deflection response, up to and including the critical load, of transversely loaded, simply supported, thin, open cylindrical shells

ABSTRACT—A description is given in this paper of part of a larger study directed toward the investigation of the buckling characteristics of thin, open cylindrical shells. The load-deflection response of transversely loaded shells is treated in this study. A subsequent paper will deal with the behavior of end-loaded open shells.

Measured values of buckling loads are compared with analytical predictions based on the presumption that prior to the onset of buckling the shell material behaves elastically but large deflections of the shell surface may occur.

Notation

\[ B = \text{half circumferential width of shell} \]
\[ B_{hy} = \text{generalized nonlinear finite-difference-equation coefficients} \]
\[ D = \text{flexural stiffness of shell} \]
\[ E = \text{modulus of elasticity} \]
\[ F = \text{stress function} \]
\[ G_{ij} = \text{gradient coefficients} \]
\[ L = \text{length of shell} \]
\[ l = \text{number of equations} \]
\[ M_{x}, M_{y} = \text{bending moments per unit width of shell strips in } x, y \text{ directions} \]
\[ M_{xy}, M_{yx} = \text{torsional moments per unit width of shell strips in } x, y \text{ directions} \]
\[ m, n = \text{number of intervals in } x \text{ and } y \text{ directions, respectively} \]
\[ p = \text{transverse load per unit projected area of shell} \]
\[ r = \text{solutions of nonlinear equations} \]
\[ R = \text{radius of cylindrical shell} \]
\[ s_{i} = \text{trial values for } r_{i} \]
\[ t = \text{shell thickness} \]
\[ u, v, w = \text{displacements in } x, y \text{ and } z \text{ directions, respectively} \]
\[ V_{x} = \text{reaction force per unit width of shell strip in } y \text{ direction} \]
\[ x, y, z = \text{longitudinal, circumferential and radial coordinates, respectively} \]
\[ x, y = \text{dimensionless } x \text{ and } y \text{ coordinates} \]
\[ w = \text{dimensionless radial displacement} \]

\[ \alpha = \text{reference length} \]
\[ \gamma = \text{membrane shearing strain} \]
\[ \epsilon_{x}, \epsilon_{y} = \text{axial strain in } x \text{ and } y \text{ directions, respectively} \]
\[ \mu = \text{Poisson's ratio} \]
\[ \epsilon = \text{error in equation } i \text{ using trial solution } s_{i} \]
\[ \sigma_{x}, \sigma_{y} = \text{membrane stresses in } x \text{ and } y \text{ directions, respectively} \]
\[ \tau = \text{membrane shear stress} \]
\[ \phi = \text{dimensionless stress function} \]
\[ \psi = \text{half subtended angle of shell} \]
\[ \nabla^{2} = \text{biharmonic operator} \]

Introduction

Cylindrical shells used in civil-engineering structures, as opposed to aircraft structures, are generally of the open type—not a complete cylinder. Theoretical studies of the buckling of this type of open shell have been undertaken previously at the Illinois Institute of Technology, and elsewhere,1-3 but little in the way of experimental results is available. An experimental study of open-shell buckling and the correlation of results with theoretical predictions of behavior is the objective of this study.

Generally speaking, the predominant stress component for a long, slender open shell with small \( t/R \) ratio is the longitudinal one and, hence, when this type of structure buckles, it does so when the longitudinal compressive normal stresses become large. The precise determination of the theoretical critical load in a shell subjected to transverse live load is, of course, an intricate analytical problem. Prior to the solution given by Chu and Turula,4,5 the buckling characteristics of a long, transversely loaded shell were considered by many to be similar to those of a curved panel subjected to axial compression, and the buckling characteristics of a short shell to be similar to those of a cylindrical tube subjected to external radial loads (c.f., Timoshenko and Gere3 and Bijlaard4). In current engineering practice, these presumed similarities of behavior are treated as approximations to be used with a “factor of safety” to obtain a design value for the critical load. This factor of safety is sometimes experimentally verified, of course.

Chu and Turula4 recently published a study in which a new technique was presented to obtain the
critical value of the lateral gravity loading for an open cylindrical shell with free longitudinal edges and simply supported ends. This new technique for determining the critical load involves the transformation of two 4th-order differential equations into a set of nonlinear finite-difference equations by applying the finite-difference module at each grid point. The resulting equations are then solved by the Newton-Raphson method as outlined by Salvadori and Baron utilizing an electronic digital computer. The experiments reported in this paper are tests performed on six open-shell structural models. The test specimens and the loads applied were so chosen as to yield results which may be directly compared with predictions based on the theoretical approach previously developed by Chu and Turula.

Theoretical Background

Figure 1 depicts a cylindrical shell of radius $R$, thickness $t$ and length $L$. The circumferential distance between the free boundaries is $2B$, and the subtended central angle is $2\phi_0$. The displacement at any point is defined in terms of components $u$, $v$, $w$ along the $x$, $y$ and $z$ coordinate axes, respectively. The two ends of the structure are simply supported and the two longitudinal edges are unsupported.

The two governing equations needed to obtain the critical gravity loading for an open cylindrical shell may be described as follows: (1) The first governing equation may be derived from a consideration of the compatibility of strains. The well-known strain-displacement relationships are:

$$ \epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \quad (1a) $$

$$ \epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 - \frac{w}{R} \quad (1b) $$

$$ \gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \quad (1c) $$

By differentiating these expressions, it can be shown that

$$ \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma}{\partial x \partial y} = \left[ \left( \frac{\partial^3 w}{\partial x \partial y^2} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2} \right] \quad (2) $$

The stress-strain relationships are:

$$ \sigma_x = \frac{E}{1 - \mu^2} (\epsilon_x + \mu \epsilon_y) \quad (3a) $$

$$ \sigma_y = \frac{E}{1 - \mu^2} (\epsilon_y + \mu \epsilon_x) \quad (3b) $$

$$ \tau = \frac{E}{1 - \mu^2} \left( \frac{1 - \mu}{2} \right) \gamma \quad (3c) $$

The membrane stresses must satisfy the equilibrium equations,

$$ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \quad (4a) $$

and

$$ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} = -\frac{p}{t} \sin \left( \frac{y-B}{R} \right) \quad (4b) $$

It is convenient to introduce a stress function $F$ defined as,

$$ \sigma_x = \frac{\partial F}{\partial y^2} \quad (5a) $$

$$ \sigma_y = \frac{\partial^2 F}{\partial x^2} + \frac{pR}{t} \left( \cos \left( \frac{y-B}{R} \right) - \cos \frac{B}{R} \right) \quad (5b) $$

and

$$ \tau = -\frac{\partial^2 F}{\partial x \partial y} \quad (5c) $$

From eqs (2), (3) and (5), the first governing equation may be obtained as,

$$ \nabla^4 F + \frac{\mu p}{R} \cos \frac{y-B}{R} = E \left[ \left( \frac{\partial^3 w}{\partial x \partial y^2} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2} \right] \quad (6) $$

Fig. 1—Shell geometry and loading pattern