Stress Analysis by Combination of Holographic Interferometry and Boundary-integral Method

Surface displacements obtained by double-aperture speckle interferometry are used in the boundary-integral method for the calculation of stresses over the region of the studied model.

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ABSTRACT—The paper describes a hybrid experimental and numerical method analysis of bodies. It consists of the experimental method of double-aperture speckle interferometry and the boundary-integral method. The interference patterns allowing evaluation of the displacement vector are obtained by the speckle interferometry. The boundary displacements obtained experimentally are conveniently used for the calculation of stresses in the body by the boundary-integral method. Some examples bear witness of the effectiveness and accuracy of the hybrid technique.

Introduction

Methods of experimental stress analysis allow one to determine stress components in the interior and on the surface of solids by the evaluation of measurements. It is, however, not possible always to obtain the displacement and stress field by experiments alone. It becomes necessary to utilize numerical methods to supplement experimental methods. The combination of the finite-element method for determination of bending moments of a plate and the experimentally obtained deflections is described in Ref. 1.

The boundary-integral-equation (BIE) method is very convenient for combinations with a number of experimental methods. The BIE method for stress analysis is based on the numerical solution of integral equations. The method is very well suited to solve two- and three-dimensional problems as it reduces them to boundary solutions, i.e., only elements on the boundary need to be defined. Depending on the completeness of the experimental data, we first study the unknown reactions and/or displacements on the boundary of the studied region or we proceed immediately to the calculation of the stress and strain field at the internal points of the region.

The hybrid numerical and experimental method for the analysis of three-dimensional problems permits the determination of stress components at interior points non-destructively. The presented method has another advantage in comparison with a purely numerical approach by allowing a reduction of the initial complicated problem to a problem with simple boundary conditions. For example, let us study the problem of vertical plate with a hole taking into account the friction between the plate and the base. The determination of the boundary conditions on the lower edge of the plate would be very complicated. By specifying the values of displacements on the boundary of the region a simple problem is obtained. The displacement values on the boundary have been found by speckle interferometry. Another case used as illustration of the numerical-experimental method is a three-point flexure of a beam including a crack. Procedure for three-dimensional problems is similar.

Measurement of Displacements by Double-aperture Speckle Interferometry

Optical methods using the property of coherent light to form laser speckle belong to the promising methods of experimental stress analysis. Their use for the analysis of deformable bodies allows measurements of displacements of individual points on a diffused surface.

To these methods belongs the double-aperture speckle interferometry (DASI) method, the principle of which has been described by Duffy. The basic element of an interferometer is a lens with a double-aperture diaphragm. The lens is placed so as to project the image of a diffused surface illuminated by coherent light on a holographic plate. The double-exposure specklegrams present lines of equal values of components of displacement vector in the surface plane. A wider use of this method in experimental practice has hitherto been prevented by a number of problems, particularly the problem of very long exposure times due to the small apertures and a generally poor quality of the fringe pattern. If the DASI method is used with transparent models with one ground screen, the exposure time may be substantially reduced. The ground screen scatters the coherent light passing through it (Fig. 1). In the given case it was shown as advantageous to use a spherical illuminating wavefront converging to a point in the plane of the image-forming lens. This is how equal luminous flow through both diaphragm apertures is secured with a view to the symmetry of the light scatter due to the ground screen and a maximum depth of modulation of the laser speckles by the grid. A high diffraction effectivity of the specklegram is reached as well. The use of the transmission technique and a He–Ne laser with 10-mW output allowed us to reduce the exposure times to as little as 10 ~ 20 s. (Agfa Gevaert 10E75 photographic plates were used.)

The interference fringe patterns were recorded by the generally used double-exposure procedure involving the deformation of the body between the first and the second exposure. The simplified relationships for the interpreta-

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tion of the fringes ignoring the out-of-plane displacements were mentioned by Duffy.\textsuperscript{a} The components of the displacement vector $u_i$ or $u_2$ parallel with the corresponding axis of coordinates and the constants of interference sensitivity depending on the spacing of both apertures of the aperture diaphragm $2d$, the distance of the object from the lens $p$ and the wavelength of the light $\lambda$.

$$u_i = N_i \frac{\lambda p}{2d}, \quad i = 1, 2 \quad (1)$$

We tried to treat the problem of improving the quality of the fringes (by quality we mean a satisfactory contrast and reduction of granularity) by selecting optimum parameters of the interferometer and the model. One of the conditions of generation of double-exposure specklegram fringe pattern is the correlation of the patterns for both exposures. It means that the displacement values of each point of the diffuse surface must not exceed the characteristic dimensions of the laser speckles. It is well known that the approximate sizes of the laser speckles in the image vary in inverse proportion with the diameter of the circular apertures of the lens diaphragm in accordance with the formula for the Airy diffraction circle. In this way, the proper choice of aperture sizes, depending on the assumed maximum values of the displacement vector, allows the minimum granularity of the interference pattern for each particular case of the measured displacement field to be obtained.

The obtained specklegrams may be reconstructed by Fourier filtering. However, due to the fact that this procedure requires a wide-open and large lens, it is advantageous to perform the filtering using only one ($+1$ or $-1$) diffraction order. In this way the reconstruction scheme becomes very close to the reconstruction of the image-plane-type holograms (Fig. 2).

The evaluation of the obtained interference patterns (isothetics) according to eq (1) yields the components of the displacement vectors $u_i$ and $u_2$ at the points on the boundary of the studied region. These values are convenient to use for calculation of stresses in the body using the boundary-element method.

**Boundary-Integral Method**

In recent years the boundary-integral method (or boundary-element method—BEM) has become a very effective analysis method for some two- and three-dimensional problems. The advantage of this numerical method consists in the reduction of dimension of the studied problem. Furthermore, in treating the problems of stress concentrations, this method gives more accurate results while saving the storage capacity and the machine time of the computer in comparison with other methods.\textsuperscript{4}

The theoretical basis of the BEM consists in replacement of the original partial-differential equation of the displacements by Somigliana's identity\textsuperscript{a,4}

$$u_i(p) = - \int T_{ij}(p, Q) u_j(Q) ds + \int U_{ij}(p, Q) t_j(Q) ds \quad (2)$$

where

$$u_i(p) = \text{the displacement vector at an internal point} \quad p = (x_i, x_j)$$

$$u_j(Q) \text{ or } t_j(Q) = \text{the value of a displacement or traction vector on the boundary of region } L$$

The kernel functions $T_{ij}(p, Q)$ and $U_{ij}(p, Q)$ represent the tractions and displacements in the $i$ direction at point $Q = (x_i, x_j)$ due to unit forces in the $j$ direction at point $p$. We can write then according to eq (2)

$$U_{ij}(p, Q) = - \frac{k}{2\pi(k + 1)G} \left[ \delta_{ij} \ln r - \frac{1}{k} r_{,i} r_{,j} \right] \quad (3)$$

$$T_{ij}(p, Q) = - \frac{1}{2\pi(k + 1)G} \left[ \{(k - 1)\delta_{ij} + 4r_{,i} r_{,j}\} \frac{dr}{dn} + (k - 1)(r_{,i} n_{,i} - r_{,j} n_{,j}) \right]$$

where

$$r^2 = (x_i - x_i)^2 + (x_j - x_j)^2$$

$$\delta_{ij} = \text{Kronecker delta}$$

$$k = \text{material parameter: } k = 3 - 4\nu \text{ for problem of plane strain and } k = \frac{(3 - \nu)}{(1 + \nu)} \text{ for generalized plane stress with } \nu \text{ being Poisson's ratio}$$

$$G = \text{shear modulus of elasticity}$$

If point $p$ converges to point $P$ on the boundary of the region (Fig. 3), eq (2) is converted to a system of integral equations expressing the relationship between the bound-