Investigation indicates that scale models can be used to show the motion of breakaway signposts and lightposts after being struck by automobiles.

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ABSTRACT—In a new approach to scale modeling, the model rules, as well as the essential $\pi$-numbers, are obtained directly from the physical laws identified as governing a phenomenon. Contrary to the widely used parameter approach with dimensional analysis, the new approach simplifies the derivation of model rules and gives a clearer physical meaning to $\pi$-numbers. As an application of the method, a study of automobile collisions in which roadside obstacles, such as signposts and lightposts, are made to break away when struck to lessen injuries is described. Both an analytical approach and full-scale testing are difficult for studying the collisions, the one because of the complexity of the phenomenon and the other because a systematic study of parameters that influence the effectiveness of a design is prohibitive in both time and cost. Laboratory tests by scale models conducted according to the established model rules were photographed with high-speed motion pictures. Agreement of the breakaway motion with that of full-scale counterparts proved the feasibility of using scale models for design improvements of such obstacles.

Symbols

- $C$ = material constant
- $E$ = modulus of elasticity
- $F$ = force
- $W$ = weight
- $g$ = gravitational acceleration
- $i$ = electric current
- $k$ = integer
- $l$ = length
- $m$ = integer
- $n$ = integer
- $q$ = physical quantity
- $t$ = time
- $v$ = velocity
- $\epsilon$ = strain
- $\theta$ = temperature
- $\rho$ = density
- $\sigma$ = stress
- $\pi$ = $\pi$-number

Subscripts

- $s$ = stress
- $g$ = gravitation
- $i$ = inertia
- $r$ = representative

Superscripts

- $^*$ = scale factor

Introduction

To reduce injuries caused when automobiles collide with fixed obstacles, road signs and lightposts are made to break away when struck. To determine an effective breakaway design, both the California Division of Highways and the Pennsylvania Department of Highways, among others, have conducted a series of field tests using full-scale vehicles and breakable wooden supports for the posts. In addition, Cornell Aeronautical Laboratory, Inc. conducted a series of tests for the New Jersey Department of Transportation using breakaway metal signposts, while both the California Division of Highways and Texas A & M University made similar studies of lightpost assemblies. Since the effectiveness of a design is influenced by a number of parameters, such as vehicle impact speed, angle of impact, relative geometry of the obstacle to the vehicle, relative debris trajectory after breakaway with respect to the vehicle path, and mounting base, many systematic tests are required before the design can be op-
timized. Thus, and especially if the reproducibility of test results is questioned, full-scale experiments become prohibitive in time and cost.

The problem, then, is to find out whether scale models can be used to show the motion of breakaway signposts and lightposts after being struck by automobiles.

**Required Conditions for Scale Modeling**

A similarity is realized between the original phenomenon (called the prototype) and a model of it if each of the prototype's quantities having a like parametric dimension can be transformed into the corresponding quantities of the model by a constant scale factor. In general, similarity requires the relation

\[ q = q'q' \]

where \( q \) and \( q' \) represent any corresponding quantities of the prototype and the model, and \( q' \) is the respective scale factor. The requirement calls for all corresponding lengths to have a constant length-scale factor,

\[ l' = \frac{l_1}{l_1'} = \frac{l_2}{l_2'} = \ldots = \frac{l_i}{l_i'} = \ldots = \frac{l_0}{l_0'} \]

all corresponding velocities to have a constant velocity-scale factor,

\[ v' = \frac{v_1}{v_1'} = \frac{v_2}{v_2'} = \ldots = \frac{v_j}{v_j'} = \ldots = \frac{v_n}{v_n'} \]

and so forth. A scale factor is the ratio of any corresponding quantities having the same parametric dimension; and it is interpreted as a ratio of representative quantities. In other words, the existence of a constant scale factor implies the existence of a representative quantity.

There are as many scale factors as physical quantities. Any physical quantity can be expressed in terms of no more than five primary quantities (usually length, time, force, temperature and electric current). Thus, if each of the five primary quantities has a constant scale factor, then any other corresponding quantities can be scaled by a combination of the five primary scale factors, and the result is a physically similar model.

These conditions sound simple, but we soon realize that they are not when we look at a practical problem. For instance, in modeling collisions of automobiles with breakaway signposts or lightposts, such forces as those created by the collision at corresponding times, such lengths as those between the car and the broken debris at corresponding times, and so on, must repeat those of the prototype on different scales of length, time and force.

**Law Approach to Establishing Model Rules**

In principle, scalings of the primary quantities can be made arbitrarily and independently of each other. A simple example to show the development of scale modeling can be found by observing a phenomenon under a magnifying glass, where every linear dimension is enlarged proportionally so that we can call it a scale model, with its behavior exactly similar to the prototype. Once we determine the scalings of the five (or fewer, if fewer will suffice to describe the phenomenon) primary quantities in similar manners, all secondary quantities will be automatically scaled by the scalings that we have chosen for the primary quantities. Note, however, that the density—a material constant—of the "model" in the above example is also scaled. Should we wish to build this large-scale model, we must use different materials, which will possess the properties specified by the chosen scalings of the primary quantities; that is, the choice of primary scale factors must involve the scaling of physical and material constants, as well.

Each material constant is defined by a physical law. For instance, Young's modulus is defined by Hooke's law of elasticity. In general, each physical law delivers an expression of scale factors of material constants, \( C' \), in terms of primary scale factors as:

\[ C' = \left( \frac{\rho}{\rho'} \right)^{n_1} \left( \frac{l}{l'} \right)^{n_2} \left( \frac{l}{l} \right)^{n_3} \left( \frac{t}{t'} \right)^{n_4} \]

where \( l' \), \( t' \), \( F' \), \( \theta' \) and \( i' \) are scale factors of length, time, force, temperature and electric current; the exponents \( n_1 \) through \( n_4 \) are typical for the given law. Identifying the material constants to be scaled is then accomplished by identifying the governing physical laws of the phenomenon.

The scaling of material constants is usually difficult, because rarely are natural or synthetic materials tailored to present us with the properties we are seeking. To discriminate between the unimportant material properties and those that must be scaled in a specific problem, we must identify the essential physical laws that govern the phenomenon. For instance, we specify that Young's modulus and density must be scaled, because we recognize the governing role of Hooke's law of elasticity and Newton's law of inertia in the given phenomenon. In this case, the material constants are to be scaled as:

\[ E' = \frac{E}{E^2} \quad \text{and} \quad \rho' = \frac{\rho}{\rho^2} \]

By eliminating \( F' \), the simultaneous relations reduce to

\[ E' = \frac{E^2}{\rho^2} \]

In many problems, however, material properties cannot be arbitrarily specified as mentioned before. Such a dilemma imposes many practical restrictions upon the choice of primary scale factors. For in-

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1. Model tests with light, such as the study of illumination in the interior design of buildings, are concerned with geometry and luminous intensity only; no model test is known to the author where luminous intensity is scaled with other primary quantities. Hence, luminous intensity is not considered here.

   There is nothing sacred about selecting which five to be the primary quantities. In astronomy, for instance, we use light-years as the measure of distance to stars. In this case, the primary quantities are speed of light and time, while distance (length) becomes a secondary quantity. As a more common example where length is secondary, we often say, "It takes so many hours by jet," to describe how far the distance is.

2. If a physical law does not involve a material constant, the law gives a contradictory relationship; that is, one of the primary quantities can be expressed by the others. Such a result warns us that our choice of primary quantities is erroneous or that we have identified a wrong physical law which only defines a secondary quantity.