Mathematical and Physical Models

A comparison of physical and mathematical models is given, showing the possibilities of physical models in connection with modern electronic equipment

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ABSTRACT—Mathematical and physical models are considered by reference to some fundamental differences; the main advantages and disadvantages of each method are emphasized. Then, the possibilities are shown, which physical models offer today with the application of modern techniques, especially when they are used in combination with digital computers, as, for example, in the so-called "hybrid-static" (hybrid structural model analysis).

Introduction

In order to describe a physical process, we must first observe how and under what conditions it occurs. In so doing, we shall ascertain qualitatively what quantities influence the process and what relations exist between them. The connections and mutually operative conditions affecting the quantities will be established which, under given circumstances, are reproducible. To allow a comparison of these observations, measurements are necessary, i.e., numerical values are allocated to the physical factors involved by standardization. This is achieved with the aid of suitable physical operations using units or comparative standard gages. The possible realizations of a physical quantity observed are mapped onto the set of real numbers by a quantitative comparison with a defined factor of the same kind, namely the comparative standard gage.

The next step is to describe the relation between the factors mathematically. First, their reciprocal effect is established by mental association. In other words, we form a mental model of the process and it is then represented by means of analysis. The analytical function finally describing the process is a mapping of the physical event onto an analogous numerical procedure. Using an allocation specification, pairs of values are allocated to each other in such a way that an analogous numerical mapping of the physical event is possible. In other words, one uses a mathematical model to describe the physical process. The word 'model' is used here to describe the mapping of one event by another, analogous event, of one thing by a similar thing; imitating or specially emphasizing its characteristic function and mode (from the Latin, modus)—hence, the word model. In representa-
tions with models, nonessential elements, or factors which do not essentially influence the nature of the process are disregarded or neglected.

However, a process under consideration can be represented by mapping it onto another physical process, that is, by a physical model. In doing so, one can even go so far as to map one process onto another physical process which has nothing more in common with the original than a formally identical mathematical description. It, however, is easier to measure than the process under consideration. In other words, the same mathematical model may be used to describe two otherwise completely different processes, mapped onto one another via the mathematical model (Fig. 1). These so-called analogy models, of which the most modern form is the electronic analog computer, will not be considered further here. From the point of view of their characteristics, these ‘analogy models’ are only a special kind of physical model. They are of no consequence for the following fundamental considerations.

Classical Mechanics

In classical mechanics, as we know, continuous functions are used to map mechanical processes. The allocation specification \( y = f(x) \) produces an infinite number of pairs of values \( x \) and \( y \), enabling the process under consideration to be mapped numerically. With complex problems, the process observed can at first only be described infinitesimally, and this leads to a set of differential or integral equations. For a real case and its specific geometry, they must be solved. Here certain difficulties arise, since the boundary conditions often do not permit an ‘analytical solution’. One then resorts to numerical approximation methods, such as series solution; in principle, these represent strict solutions of the analytical problem, since they converge on the exact solution. The approximation is the result of limiting to a finite number of algebraic operations. The use of digital computers usually makes very good approximations possible. However, in order to satisfy complicated boundary conditions, a great deal of thought is consistently necessary. In each case a solution has to be found which is adapted to the problem, and this is often very laborious. Realistic boundary conditions can hardly be found with a mathematical model. Here, mapping of the process under investigation onto a physical model is always superior. However, only a functional representation derived from the mathematical models of classical mechanics is capable of making universal statements about the qualitative and quantitative progression of a process and of a whole group of problems. A representation of this kind contains more than merely the description of an individual case in application. Thus, it is of great technical significance for the dimensioning and optimization of a structure at the design stage. In addition, it enables the experimental results obtained from measurements on physical models to be assessed and classified. Last but not least, it supplies the fundamentals for the construction of physical models according to the theory of models and dimensional analysis, from which the structural laws to be observed are derived.

Finite Elements

The method of finite elements is quite different. Admittedly, it is also a mathematical mapping of mechanical processes but, in this case, one confines oneself even in principle to a finite number of pairs of values of the function describing the process.

In practice, the structure in question is divided into many simple elements. For instance, in order to solve a plane-stress problem, the continuum is divided into simple elements, e.g., triangles (Fig. 2) connected to each other only at their corners, the so-called nodes. In doing so, one confines oneself consciously to a finite number of elements of finite size. First, one observes the linear-elastic response of the individual element. For the forces stressing it at its corners and the deflections possible here a relationship is postulated which connects these factors via the rigidities and the geometry of the element. In doing so, it is essential that the elastic response within the element can be described unequivocally by stating the deflections or forces at its corners, the nodes. For, if one now compiles a random structure from such elements, the analysis of its elastic response under the influence of external forces or deflections may be explained by a system of linear equations (Fig. 3). Approximation is already extant in this method because, for one thing, the continuum has only been calculated approximately by division into elements. In addition, due to mandatory simplifications in describing the elastic response of the elements, not all compatibility conditions between the elements can be fulfilled simul-