A Multidimensional-crack-growth Prediction Methodology for Flaws Originating at Fastener Holes

Predicting the growth of flaws from fastener holes requires a methodology which accounts for residual stresses due to yielding at the hole edge as well as the changing shape of the flaw.

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ABSTRACT—A multidimensional-crack-growth prediction methodology for flawed fastener holes is described. The methodology uses a slice-synthesis model of the flawed hole for predicting the crack-driving force around the periphery after each increment of advance. Analytic predictions of the crack-growth rate are compared with test results for various examples. It is shown that residual stresses due to yielding at the hole edge and the cycle-by-cycle shape changes are a major cause of apparent crack-growth retardation or acceleration, and that the assumption of a constant shape, as is a common practice, can lead to highly misleading estimates of service life.

Introduction

During the past decade, fracture mechanics has become an established discipline for designing damage-tolerant aircraft structures. Analytical techniques have been developed which permit the fracture specialist to predict the service life of structures containing manufacturing or service-induced flaws using coupon laboratory data and stress analysis. Usually, these techniques model the crack-growth process as one-dimensional in order to minimize computational difficulties.

However, the most frequent problem in aircraft damage-tolerant design—that of flaw growth from a fastener hole—invariably involves a two-dimensional pattern of growth. Fractographic examination of the crack-growth pattern for these flaws indicates that they grow in either of two ways: as a corner flaw or as an embedded flaw. As shown in Fig. 1, these flaws have a semielliptical shape until the critical dimension is exceeded or the crack breaks through the back surface and grows as a through-the-thickness crack. In either case, the majority of crack-growth life occurs in a two-dimensional pattern with the shape changing due to the complex interaction between crack geometry, loading history and material properties.

Predicting the service time required to propagate such a crack from its initial size to critical size is a difficult task. Even at the simplest continuum-mechanics level, a complete description of the crack-growth process requires (1) a three-dimensional stress-analysis capability for characterizing the distribution of crack-driving force along the crack front at any stage of growth, (2) an incremental-growth model which relates cycle-by-cycle shape changes to the global rate of growth, and (3) an ability to account for the alteration of the crack-driving force due to residual plastic strains resulting from yielding at the hole edge or at the crack tip.

Most methods now in use for predicting crack growth from fastener holes do not explicitly account for all of these factors. The crack is modeled as a through-the-thickness crack with crack-front curvature effects introduced through the so-called 'shape factor'. By specifying flaw shape a priori, it is necessary only to track growth at a single point on the crack front; load-interaction effects on crack growth are handled through such phenomenological one-dimensional crack-growth models as the Wheeler or Willenborg models or analytically through models based on the crack-closure phenomenon.

Various degrees of success have been achieved using these techniques. But as the assumption of a quasi one-dimensional growth pattern does not reflect the actual mechanisms, successful use has invariably required 'fine tuning' through empirical parameters to make the predictions agree with observed test results. Stone and Swift recognized this shortcoming in their paper on a future damage-tolerance approach to airworthiness certification and proposed a two-dimensional crack-growth prediction methodology based on stress-intensity factor solutions in Ref. 7 for common flawed-hole geometries. These solutions are valid only for cases in which no macroscopic yielding occurs at the edge of the hole; consequently, the methodology does not account for any shape changes induced by residual stresses resulting from this yielding.

In this paper, a general two-dimensional crack-growth prediction methodology, which accounts for the effects of these stresses, is described and analytic predictions of crack-growth behavior are compared with test results. The methodology is based on the use of a slice-synthesis model of the flawed hole for predicting the stress-intensity-factor distribution along the periphery of the crack after each increment of advance. The simplicity of
the slice-synthesis approach permits solutions for complex flaw geometries to be obtained ‘on-line’ during program execution. This stress-analysis capability is used to predict the variation of the stress-intensity factor around the periphery. A differential-growth-rate law is used to determine the increment-by-increment (or cycle-by-cycle) changes in the shape of the crack. Elasto-plastic stress analysis of the unflawed member is used to predict the effects of residual stresses induced at the edge of the hole on the global pattern and rate of flaw growth. The effects of crack closure due to the cyclic accumulation of plastic strains in the wake of the advancing tip are incorporated using an approach used by Dill and Saif to predict the growth of surface flaws which undergo shape changes.

Various examples are presented to illustrate the importance of incremental shape changes. These are (1) a crack growing from an open or loaded hole in the absence of plastic yielding at the hole edge, (2) the effect of a tensile overload on crack growth, and (3) the growth of a corner flaw in a lug subjected to spectrum loading. The good agreement between test results and predicted behavior verifies the important role that the shape-change phenomenon plays. It is found that the usual assumption of a constant shape can lead to significant underestimation of the stress-intensity factors when no plastic yielding occurs at the hole. When such yielding occurs, the assumption of a constant shape is likewise erroneous as the crack-shape-change effects, together with the reduction in crackline loading due to yielding, are a major mechanism of crack-growth retardation. This fact is of significance since many one-dimensional crack-growth prediction schemes implicitly assign all retardation to load-interaction effects alone and fail to account for these other factors.

The Slice-Synthesis Method

In order to predict the incremental-growth pattern of a flaw, it is necessary to have a three-dimensional stress-analysis capability which can provide an updated estimate of the variation of stress-intensity factor along the front after each increment of growth. The slice-synthesis method permits three-dimensional solutions to be obtained using conventional two-dimensional techniques. Such powerful and well established two-dimensional analysis techniques as complex-function theory and elastic reciprocity, i.e., the universal weight function of Rice and Bueckner, can be extended to three-dimensional geometries, thereby making a complete analysis economically possible.

The approach will be illustrated for the corner flaw, Fig. 2. The embedded flaw can be formed by extending the corner flaw, through its mirror image, across the plane formed by the x-y axes. The slice idealization of the solid is shown in Figs. 3 and 4. In Fig. 3, the solid is idealized as a series of slices oriented in the xy plane. This results in through-the-thickness radial crack elements stacked layer-by-layer over elements containing an unflawed hole. The length of the crack for any slice located a distance z below the free surface of the plate is \( a(z) \) while the vertical distance between the tip of the crack and the top surface of the plate is \( b(x) \). Mechanical coupling between the slices arises from the transverse-shear stresses \( \tau_{y}(x,y,z) \) and \( \tau_{z}(x,y,z) \) which exist on any xy plane of the actual solid. However, \( \tau_{y}(x,y,z) \) has only a minor influence and can be ignored, leading to the idealization shown in Fig. 3(b). The influence of \( \tau_{z}(z,y,z) \) is simulated by a system of line springs attached to the faces of the crack as shown in Fig. 3(c). These springs effectively restrain crack opening in the same manner as the interlaminar-shear stresses.

The properties of these springs are determined by idealizing the solid per Fig. 4 as a series of slices each of which is a single-edge crack element. It is assumed that no direct mechanical coupling exists between these slices, each of which is in a state of generalized plane stress set up by a compressive boundary stress \( -p^{*}(x,z) \) acting on the faces of the flaw formed by the continuous distribution of these elements of infinitesimal thickness. This stress system is identical to but opposite in sign to the tensile stress \( p^{*}(x,z) \) transmitted across the springs bridging the faces of the radial crack slices.