Displacement Field Around a Circular Hole in a Viscoelastic Plate

Experimental verification is provided to show that the solution to statically determinate plane viscoelasticity problems can be closely approximated by a series of quasi-static states using known elastic solutions.

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ABSTRACT—Moiré experimental techniques are used to measure displacement fields in viscoelastic plates undergoing large deformations at elevated temperatures. These experimental procedures are applicable to determining displacement fields in nonlinear materials.

As preliminary information, the material properties are determined from creep studies. The moiré method is used to determine the strains under constant load and isothermal conditions. Tests are conducted for several combinations of load and temperature for 2.5 decades of time. Assuming thermorheologically simple behavior, the data are shifted to establish the creep extensional compliance over ten decades in time.

The constitutive equations are formulated as integral equations, the kernels of which are the functions that were measured in this work. These equations are solved exactly for the infinitesimal case. The finite case is then approximated by an incremental superposition of a series of successive infinitesimal solutions. The results are applied to a plate initially containing a circular hole, and are shown to agree closely with the experimental measurements.

Nomenclature

\[ E,\nu \] = elastic constants
\[ F(t),\psi(t) \] = viscoelastic creep-compliance functions in extension
\[ G(t),K(t) \] = shear and bulk moduli functions
\[ \delta_{ij} \] = Kronecker's tensor
\[ H(t) \] = Heaviside function
\[ S_{ij},e_{ij} \] = stress and strain deviators
\[ \sigma_{ij}(x,t),\epsilon_{ij}(x,t) \] = viscoelastic stress and strain tensors
\[ T \] = tensile stress
\[ x,y \] = coordinates in longitudinal and transverse directions
\[ z = x + iy \]
\[ u,\nu \] = elastic-displacement components in \( x,y \) direction
\[ u(x,t),v(x,t) \] = viscoelastic displacements in \( x,y \) direction
\[ a,b \] = semimajor and semiminor axes of ellipse
\[ \xi,\eta \] = coordinates in transformed plane corresponding to \( x,y \) directions in physical plane
\[ f = \xi + iy \]

\[ z = u(\gamma) = R[\gamma + (m/\gamma)] \]
\[ R = (a + b)/2 \]
\[ m = (a - b)/(a + b) \]

Introduction

The distinguishing feature of viscoelastic media lies in the time-dependent stress-strain relationship. Earlier works derived various constitutive laws employing idealized viscoelastic models, which can be represented by various spring and dashpot combinations. Unfortunately, these models cannot predict viscoelastic solutions to realistic materials unless large numbers of elements are used in the model. Such large-element models soon become mathematically awkward and create extremely tedious algebraic calculations. Recently, numerical approximation techniques have been developed to employ broadband realistic characterization data in conjunction with the elastic-viscoelastic analogy. In this way, realistic behavior can be incorporated into the solution and the assumptions associated with idealized model behavior are avoided.

An alternate method of formulating viscoelastic problems consists of using hereditary integral equations, the kernels of which may be determined experimentally. Lee and Rogers developed a finite difference procedure to solve viscoelastic problems using this formulation.

More recently, Schapery developed a successive approximation scheme by rearranging the hereditary integrals in terms of the differences between the viscoelastic problem and a fictitious elastic

Fig. 1—Testing chamber

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Fig. 2—Tensile specimen and time sequence of moiré patterns for material characterization

problem. This method will be used as a basis for the theoretical solution to the problem in this paper. Furthermore, it will be shown that, if the stresses are statically determinate, the iterative procedure described by Schapery is not necessary. In these problems, the solution is simply the elastic solution with the elastic constants replaced by their corresponding viscoelastic moduli functions.

If very large deformations are present, the solution can be expressed in terms of a series of quasi-static states. Thus, the memory dependence of the viscoelastic material may be included by considering a series of similar fictitious elastic problems.

Experimental viscoelastic analyses, particularly at elevated temperatures, have lagged the theoretical analyses considerably. The moiré method of determining displacements is ideally suited for plane viscoelastic problems since the displacements are large. Although multidimensional viscoelastic measurements are not common in the literature, some measurements have been reported by Dantu to demonstrate feasibility of the method, by Daniel to characterize polyvinylchloride, and by Hart to show applications in propellant-grain materials.

Experimental Analysis

Equipment was designed so that specimens could be photographed while being loaded in uniaxial tension in a constant ambient temperature. A schematic of the test chamber is shown in Fig. 1. The chamber was heated by forced air. Insulation completely surrounded the chamber except for the viewing windows which had an independent heat source to simulate more accurately the desired constant-temperature condition. The temperature was controlled to within 1°F and could be maintained over long periods of time. The specimens were horizontally loaded and an oil layer was provided between the specimen and the master moiré grid to minimize friction.

The material was chosen to be polymethylmethacrylate and is commonly known as Plexiglas. There are different grades of Plexiglas and the II-UVA grade was used throughout these tests because it is initially stress-free. The commercial grade is Plexiglas G which does not have this property, and will contract at no load if subjected to elevated temperatures.

Constant-load conditions ranging from 10 to 40 lb in isothermal ambients ranging from 190 to 250°F were studied. During each test, a permanent record of the deformation was made by taking photographs at approximately equal intervals on a log time scale.

The viscoelastic material properties were determined from creep tests on unweakened tensile specimens (see Fig. 2). These specimens were photographically preprinted to have two perpendicular line arrays of 300 lines/in. These arrays were used to determine both the longitudinal and transverse displacement fields simultaneously.

To increase the accuracy, the master longitudinal array established a 2.9 percent tensile mismatch at 220°F while the transverse array established a 3.1 percent compressive mismatch at 220°F. The mismatch is due, in part, to the deliberate difference in pitch in the master and model arrays and, in part, due to the difference in thermal expansion of the master and model materials. Therefore, the mismatch was slightly different at other temperatures. By referring all measurements to the time zero condition for a given test, the correction for the mismatch was incorporated into the solution in the usual manner. The change of pitch of the glass