Whole-field Optical Examination of Cylindrical Shell Deformation

by S. Krishnakumar and C.G. Foster

ABSTRACT—In this paper an optical technique for the measurement of radial deformation in circular cylindrical shells is discussed. The technique is a modification and improvement on an earlier method, using a conical mirror of simple geometry to view a grating reflected from the inner surface of the shell. The new system allows more precise alignment of the components of the optical system with the shell. Fringes obtained by superposition as in the Ligtenberg moiré method, or deviations of lines from a regular pattern in a photographic image provide a measure of the slope changes on the surface of the shell. Theoretical relations are presented for three grid orientations, at least two of which have to be used in conjunction to determine the two components of slope. Illustrations for the use and accuracy of the technique are presented for two cases. In the first, deflections due to a tilt of the axis of the test shell are measured; in the second, deformations associated with a radial point load applied at the free end of a cylindrical shell with one end built in are determined.

List of Symbols

\[ D = \text{distance of the camera from the base of the conical mirror} \]
\[ D_0 = D - R_0 \tan (\beta) \]
\[ L_s = \text{maximum shell height} \]
\[ L_g = \text{length of the grid required for a shell of length } L_s \]
\[ n_a, n_c, n_s = \text{fringe orders for the axial, circumferential and spiral grids, respectively} \]
\[ p_a = \text{pitch of the lines on the axial grid} \]
\[ (= 1.206 \text{ mm}) \]
\[ p_c = \text{pitch of the lines on the circumferential grid} \]
\[ (= 5.85 \text{ mm}) \]
\[ p_s = \text{normal pitch of the lines on the spiral grid} \]
\[ (= 1.184 \text{ mm}) \]
\[ p_{ai}, p_{ci}, p_{si} = \text{pitches of the lines on the images of the axial, circumferential and spiral grids, respectively} \]
\[ R_a = \text{radius of the cylindrical grids} \]
\[ (= 9.6 \text{ mm}) \]
\[ R_0 = \text{base radius of the conical mirror} \]
\[ (= 74.5 \text{ mm}) \]
\[ R_i = \text{inner radius of the shell} \]
\[ r = \text{radial distance of the point of reflection on the conical mirror} \]
\[ S_a, S_c, S_s = \text{relative shifts of the incident ray on the grid surface in the axial, circumferential and the direction normal to the spiral grid lines, respectively} \]

Introduction

The photorelective moiré method introduced by Ligtenberg\(^1\) essentially consists of superimposing the image of a line grating reflected by a deformed plate over the image from the undeformed plate to yield fringes that are contours of constant changes in slope over the plate. The most appealing feature of this technique is that it is a whole-field method; that is, it provides information regarding changes in slope over the whole area of the plate. The technique has been extended to obtain slope changes over cylindrical shell sections.\(^2\) An adaptation of the Ligtenberg technique for the measurement of radial deformations in complete cylindrical shells has been reported.\(^3\) Central to that technique is the use of an internally reflective conical mirror to convert the cylindrical coordinates of the shell to flat polar coordinates suitable for photographing. The present method is a further modification of this technique using an externally reflective conical mirror. The main advantages are that the new system allows the use of a much smaller conical mirror and at the same time facilitates easier alignment of the components of the optical system with the shell.

The Experimental Setup

The experimental setup is shown in Fig. 1. The conical mirror situated at the bottom end of the shell provides a complete view of the inner surface. The image is viewed by a camera positioned in front of the loading frame via a 45-deg plane mirror positioned at the top. Shells with a highly reflective inner surface are made from epoxy resin using a spin-casting technique described elsewhere.\(^4\) The lines on the cylindrical grid positioned on the axis of the shell are reflected by the shell inner surface, the conical
mirror and the 45-deg mirror in succession. A grid with circumferential lines transforms to concentric circles in the image plane and axial lines are seen as a radial pattern. A grid with spiral lines is in essence a combination of the other two and transforms to an image also with spiral lines. The conical mirror is manufactured by polishing the machined outer surface of an aluminum (No. 2011) cone and has a base angle of nine deg. The shell is supported on an end ring at the bottom with a clearance fit to the outside of the conical mirror while the grid is supported on a cylindrical rod running through a hole at the center of the cone. With this arrangement, the grid, the conical mirror and the shell are automatically aligned along the same axis. Also the height of the grid may be varied by raising or lowering the support rod. The maximum length of the shell that can be examined may be altered by using conical mirrors with different base angles.

This arrangement is a considerable improvement over the previously described arrangement in which the inside of the shell was viewed by reflection from the polished inner surface of a large hollow cone. In this earlier configuration, since the hollow conical mirror had to be positioned some distance away from the shell at the opposite end to the grid, accurate alignment of the three components along the optical axis was difficult to achieve. Moreover the length of the hollow cone required was nearly twice that of the shell, and the minimum diameter required was about one-third the length of the shell. In comparison the conical mirror in the present system is compact and uses the more readily polished outer surface for reflection.

The grid is illuminated by a circular array of lights positioned on the crosshead of the loading frame. Vertical slits in front of the light sources direct their beams radially towards the grid, so that the shell is illuminated only by the light scattered from the cylindrical-grid surface. The view from the camera is a diverging conical field, though a collimated field could be attained by inserting a large diameter convex lens in the optical path between the camera and the loading frame. Theoretical relations have been derived on the basis that the light from the conical mirror converges to a point at the aperture of the camera. It was found necessary to take this divergence into account in the analysis; although the maximum angle of divergence was only about one deg with the camera placed at a distance of 14 feet, it was not negligible when compared to the nine-deg base angle of the conical mirror.

The sharpness of the image depends on the film speed, aperture and the distance of the camera from the shell. Since the requirement is that all parts of the grid and the shell surface be in focus simultaneously, a considerable depth of field is necessary. In the present system the aperture setting was f22 on a Pentax K1000 camera with a 400-mm focal length telephoto lens attached. The total distance between the camera and the conical mirror was 4267 mm (14 ft). The film used was Kodak technical pan film 2415 (125 ASA) which required an exposure time of about eight seconds. The minimum pitch of the grid lines is controlled by the quality of the inner surface of the shell. The grid-line spacings were chosen as the minimum suitable for a spun-cast epoxy surface.

**Geometry of the Optical System**

The optical system is illustrated in Fig. 2. It may be noted that for a ray to arrive at the camera at an angle of $\gamma$ with respect to the optical axis, it has to leave the grid at an angle of $(2\theta + \gamma)$. Also, for a given shell radius the highest section of the shell that could possibly be viewed in the conical mirror is that which could be reflected from the apex of the conical mirror. This height is denoted by $Z_0$. The following relations are readily obtained from the geometry of the figure.

\[
Z_0 = R_a \tan \theta + R_s \cot 2\theta
\]

\[
\tan \gamma = \frac{r}{D_o + r \tan \theta}
\]

where $D_o = D - R_a \tan \theta$.

\[
r = \frac{(Z_o - Z_s) \sin 2\theta}{1 + [(Z_o + (R_s - R_a) \tan \theta)/D_o]}
\]

and

\[
Z_s = Z_o - \frac{r}{\sin 2\theta} \left[1 + \frac{R_s - R_a}{r + D_o \sin 2\theta}\right]
\]

\[
Z_e = Z_o + (R_s - R_a) \cot 2\theta - \frac{r}{\sin 2\theta} \left[1 + \frac{2(R_s - R_e - R_a)}{r + D_o \sin 2\theta}\right]
\]

If the optical arrangement is modified so that only rays parallel to its optical axis are utilized, in which case $\gamma = 0$, or if the distance $D$ between the camera and the conical mirror is very large compared to the radius of the mirror so that $\gamma$ is negligibly small, then the terms containing $D_o$ in the denominator in eqs (3)-(5) vanish and these relations become linear.

The cylindrical coordinates $(Z_s, \varphi)$ and $(Z_e, \varphi)$ of the grid and shell surfaces are transformed into flat polar coordinates $(r, \varphi)$ on the image plane. While there is no change in the angular coordinate $\varphi$, the relationships between $r$ and $Z_s$, $Z_e$ are given by eqs (3)-(5). The relation

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**Fig. 1—The experimental setup**