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Particle Creation by Black Holes

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Abstract. In the classical theory black holes can only absorb and not emit particles. However it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{h\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_\odot}{M}\right)$ °K where κ is the surface gravity of the black hole. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance: any primordial black hole of mass less than about 10^{15} g would have evaporated by now. Although these quantum effects violate the classical law that the area of the event horizon of a black hole cannot decrease, there remains a Generalized Second Law: $S+\frac{1}{4}A$ never decreases where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons. This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy. It is tempting to speculate that this might be the reason why the Universe contains so much entropy per baryon.

1.

Although there has been a lot of work in the last fifteen years (see [1, 2] for recent reviews), I think it would be fair to say that we do not yet have a fully satisfactory and consistent quantum theory of gravity. At the moment classical General Relativity still provides the most successful description of gravity. In classical General Relativity one has a classical metric which obeys the Einstein equations, the right hand side of which is supposed to be the energy momentum tensor of the classical matter fields. However, although it may be reasonable to ignore quantum gravitational effects on the grounds that these are likely to be small, we know that quantum mechanics plays a vital role in the behaviour of the matter fields. One therefore has the problem of defining a consistent scheme in which the space-time metric is treated classically but is coupled to the matter fields which are treated quantum mechanically. Presumably such a scheme would be only an approximation to a deeper theory (still to be found) in which space-time itself was quantized. However one would hope that it would be a very good approximation for most purposes except near space-time singularities.

The approximation I shall use in this paper is that the matter fields, such as scalar, electro-magnetic, or neutrino fields, obey the usual wave equations with the Minkowski metric replaced by a classical space-time metric g_{ab}. This metric satisfies the Einstein equations where the source on the right hand side is taken to be the expectation value of some suitably defined energy momentum operator for the matter fields. In this theory of quantum mechanics in curved space-time there is a problem in interpreting the field operators in terms of annihilation and creation operators. In flat space-time the standard procedure is to decompose

the field into positive and negative frequency components. For example, if ϕ is a massless Hermitian scalar field obeying the equation $\phi_{;ab}\eta^{ab}=0$ one expresses ϕ as

$$\phi=\sum_i\{f_i\boldsymbol{a}_i+\bar{f}_i\boldsymbol{a}_i^\dagger\} \tag{1.1}$$

where the $\{f_i\}$ are a complete orthonormal family of complex valued solutions of the wave equation $f_{i;ab}\eta^{ab}=0$ which contain only positive frequencies with respect to the usual Minkowski time coordinate. The operators $\boldsymbol{a}_i$ and $\boldsymbol{a}_i^\dagger$ are interpreted as the annihilation and creation operators respectively for particles in the ith state. The vacuum state $|0\rangle$ is defined to be the state from which one cannot annihilate any particles, i.e.

$$\boldsymbol{a}_i|0\rangle=0 \qquad \text{for all } i\,.$$

In curved space-time one can also consider a Hermitian scalar field operator ϕ which obeys the covariant wave equation $\phi_{;ab}g^{ab}=0$. However one cannot decompose into its positive and negative frequency parts as positive and negative frequencies have no invariant meaning in curved space-time. One could still require that the $\{f_i\}$ and the $\{\bar{f}_i\}$ together formed a complete basis for solutions of the wave equations with

$$\tfrac{1}{2}i\int_S(f_i\bar{f}_{j;a}-\bar{f}_jf_{i;a})d\Sigma^a=\delta_{ij} \tag{1.2}$$

where S is a suitable surface. However condition (1.2) does not uniquely fix the subspace of the space of all solutions which is spanned by the $\{f_i\}$ and therefore does not determine the splitting of the operator ϕ into annihilation and creation parts. In a region of space-time which was flat or asymptotically flat, the appropriate criterion for choosing the $\{f_i\}$ is that they should contain only positive frequencies with respect to the Minkowski time coordinate. However if one has a space-time which contains an initial flat region (1) followed by a region of curvature (2) then a final flat region (3), the basis $\{f_{1i}\}$ which contains only positive frequencies on region (1) will not be the same as the basis $\{f_{3i}\}$ which contains only positive frequencies on region (3). This means that the initial vacuum state $|0_1\rangle$, the state which satisfies $\boldsymbol{a}_{1i}|0_1\rangle=0$ for each initial annihilation operator $\boldsymbol{a}_{1i}$, will not be the same as the final vacuum state $|0_3\rangle$ i.e. $\boldsymbol{a}_{3i}|0_1\rangle\neq 0$. One can interpret this as implying that the time dependent metric or gravitational field has caused the creation of a certain number of particles of the scalar field.

Although it is obvious what the subspace spanned by the $\{f_i\}$ is for an asymptotically flat region, it is not uniquely defined for a general point of a curved space-time. Consider an observer with velocity vector v^a at a point p. Let B be the least upper bound $|R_{abcd}|$ in any orthonormal tetrad whose timelike vector coincides with v^a. In a neighbourhood U of p the observer can set up a local inertial coordinate system (such as normal coordinates) with coordinate radius of the order of $B^{-\frac{1}{2}}$. He can then choose a family $\{f_i\}$ which satisfy equation (1.2) and which in the neighbourhood U are approximately positive frequency with respect to the time coordinate in U. For modes f_i whose characteristic frequency ω is high compared to $B^{\frac{1}{2}}$, this leaves an indeterminacy between f_i and its complex conjugate $\bar{f}_i$ of the order of the exponential of some multiple of $-\omega B^{-\frac{1}{2}}$. The indeterminacy between the annihilation operator $\boldsymbol{a}_i$ and the creation operator $\boldsymbol{a}_i^\dagger$ for the