Thermohydrodynamics of the Ocean

The study of tidal currents in the seamount area in a continuously stratified ocean*

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Abstract — In the framework of the linear theory and without using the hydrostatics approximation, we study the wave motions produced by a barotropic tide impinging upon a bottom topography feature in a continuously stratified ocean. Numerical techniques are used to estimate the effects of the stratification and the Coriolis parameter on the tidal flows in the seamount area.

This paper is supposed to be a sequel to previous investigations [1-3] concerned with the study of the spatial structure of baroclinic tides, flowing over solitary elevations of the seabed, through the use of numerical modelling techniques. In these papers, some peculiarities of the field of vertical motions and the impact of stratification parameters and of the geographical latitude, where the considered feature of the bottom topography is located, were analysed. The computations were performed for a model with a discontinuity of density [2] and a piecewise-constant profile of the Brunt–Väisälä frequency [3].

The present paper reports on the tidal current field involving a complex distribution of the Brunt–Väisälä frequency. The mathematical model applied in this study represents a generalization of the numerical algorithm given in ref. [3], designed to be used in the case of an arbitrary steady mean density distribution in the pycnocline layer.

A horizontally-infinite, continuously stratified ocean of constant depth accommodating a localized feature of seamount type is considered. A plane barotropic tidal wave from a constant depth area is running onto the obstacle. It is necessary to calculate the field of the wave velocity perturbations in the mountain area and beyond it using the given shape of the feature, the parameters of the impinging wave, and the Brunt–Väisälä frequency profile. In a linear approximation, this problem is reduced [4] to solving a differential equation for a complex pressure amplitude $p(x, y, z)$:

$$\nabla^2 p + \frac{\partial}{\partial z} \left( \alpha \frac{\partial p}{\partial z} \right) = 0$$  \hspace{1cm} (1)

with the boundary conditions

$$\frac{\partial p}{\partial z} = \beta p \quad \text{at} \quad z = 0,$$  \hspace{1cm} (2)

$$\alpha \frac{\partial p}{\partial z} + \nabla h \nabla p + i\gamma J(p, h) = 0 \quad \text{at} \quad z = -H_0 + h(x, y).$$  \hspace{1cm} (3)

The $z$-axis here is directed vertically upward; the plane of the horizontal coordinates $(x, y)$ coincides with the undisturbed free sea surface $\alpha(z) = (\omega^2 - N^2(z))^{-1}(\omega^2 - f^2)$;

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\[ \beta = g^{-1}(\omega^2 - N^2(z)), \quad \gamma = \omega^{-1} f, \quad N(z) = (-g\rho_0^{-1} \frac{dp}{dz})^{1/2} \]

is the Brunt–Väisälä frequency; \( \beta(z) \) is the undisturbed water density state; \( \rho_0 = \text{constant} \), is the mean density; \( \omega \) is the frequency of fluctuations; \( f \) is the Coriolis parameter; \( \nabla = \{\partial/\partial x, \partial x/\partial y\} \); \( J(a, b) \) is the Jacobian of the \((x, y)\) variables; \( H_0 \) is the mean depth of the ocean; \( h(x, y) \) is the bottom topography; and \( g \) is the acceleration of gravity. The wave velocity components are expressed through \( p \) by the formulae

\[ u = -M^{-1} \left\{ i\omega \frac{\partial p}{\partial x} - f \frac{\partial p}{\partial y} \right\} e^{-iw t}, \quad v = -M^{-1} \left\{ i\omega \frac{\partial p}{\partial y} + f \frac{\partial p}{\partial x} \right\} e^{-iw t}, \]

\[ w = \frac{i\omega}{(N^2(z) - \omega^2)\rho_0} \frac{\partial p}{\partial z} e^{-iw t}, \]

where \( M = (\omega^2 - f^2)\rho_0 \).

For pre-inertial period oscillations (\( \omega > f \)) — and it is these that are to be examined — the sign of the coefficient \( \alpha(z) \) in equation (1) depends on the ratio between \( \omega \) and \( N(z) \). If \( \min N > \omega \), then \( \alpha < 0 \), and equation (1) belongs to the hyperbolic type; this calls for solving a problem which in the given formulation is characteristic of elliptical equations. In the general case, this may necessitate imposing limitations on the boundary conditions and the seabed geometry. In fact, the variable \( z \) in equation (1) is similar to the time coordinate, \( t \). Therefore, to formulate the boundary value problem correctly for determining \( p \) (the Cauchy problem), it would be reasonable to have two boundary conditions for the seabed plane and no boundary conditions at \( z = 0 \). Besides, the bottom topography should not replicate characteristic surfaces of equation (1) which is not, generally speaking, excluded. In view of this, we will consider below the case which implies that the seamount is fully submerged in a weakly stratified oceanic layer, i.e., function \( N(z) \) satisfies the condition

\[ \max_{z} N(z) < \omega \quad \text{at} \quad z < \max_{(x,y)} h(x, y) - H_0. \]

A numerical algorithm is constructed on the assumption that the wave perturbations decay with range and at some distance \( r_0 \) from the top of the mountain (this distance is determined by preliminary calculations) they become negligibly small compared with the wave speed of an on-running barotropic tide. This enables us to define the horizontal dimensions of the area under consideration and to assume that

\[ p e^{-iw t} = p_0 \quad \text{at} \quad z^2 + y^2 > r_0^2, \]

where \( p_0 = p_1(z) \exp[i(kz + ly - \omega t)] \) is the known pressure distribution in the progressing wave. The rectangular-shaped area being studied is sliced into two layers in a manner which allows us to observe condition (6) in the lower stratum. In addition, the curvilinear coordinate system is applied to the deeper layer. The initial boundary value problem is approximated using the finite-difference technique. Difference schemes are described in detail in refs [1-3]. We will restrict ourselves to considering the problem of constructing a stable difference scheme for determining \( p \) in the upper layer, because specifically this makes the suggested technique differ from the one applied in refs [1-3]. The specific nature of our numerical scheme is related to the fact that equation (1) belongs to the mixed type. To derive stable algorithms for solving such equations, we use a general approach [5]. This involves the use of finite differences, depending on the type of