STRUCTURAL MECHANICS

EQUATIONS OF THE OSCILLATIONS OF A DEEP MASSIVE FOUNDATION FLEXIBLY RESTRAINED IN THE GROUND

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A new equation is derived for the rotational oscillations of a deep massive foundation flexibly restrained in the ground. An error in the corresponding equation published in the literature is indicated.

Equations of the oscillations of a deep massive foundation flexibly restrained in the ground were derived by Glushkov [1]; in that case, however, an error was permitted. He also repeats this same solution with the error in [2]. The purpose of the present paper is to derive precise equations with indication of the error in [1, 2].

Dynamic Model. The notations from [1, 2] are used for comparison. A massive foundation in the form of a parallelepiped (sides a, b, and h) with a depth (h ≥ 2a), which is fixed in the ground, is examined. The top side of the foundation is at the level of the platform beneath the dynamic load (ν, P) (Fig. 1).

It is assumed that: 1) the foundation is a rigid body; 2) oscillations occur in the vertical plane; and, 3) the soil is an elastic medium with no mass or damping. The premise concerning the soil’s elasticity is only partially justified in connection with its nonlinear properties, but is nevertheless employed in engineering practice.

The following notations are adopted in the paper: O is the center of gravity; X and Y are the principal central axes; x and y are the displacements of the center of gravity; φ is the turn angle; ρ is the density, M is the mass (M = abhp), and θ₀ is the moment of inertia about the axis through O (θ₀ = M(a² + h²)/12).

The in-plane oscillations of the foundation can be separated into vertical oscillations that are independent (see Fig. 1), and also mutual horizontal and rotational oscillations (Fig. 2). Both the inertial forces (M₂, M₅, and θ₀φ), and the elastic reactions of the soil (R₁, R₂, R₃, R₄, and R₅) and their moments (M₂, M₃, M₄, and M₅) are in opposition to the dynamic load (V, P, m = Va₁ + Ph₁) at any time t.

Using the lower-surface factor c [Fuss, 1798; Winkler, 1867] for uniform (c₂, c₃) and nonuniform (cₜ, c₄) compression, corresponding elastic reactions develop directly from the proposed reactive pressures of the soil (see Figs. 1 and 2). For a lateral pressure (see Fig. 2) described by the parabola f₁, for example,

\[ R₄ - R₅ = \frac{cₐh^2bφ}{48} - \frac{5cₐh^2bφ}{48} = \frac{cₐh^2bφ}{12}. \]  

(1)

Then, considering the arms (r₄ = h/4 and r₅ = h/20),

\[ M₄ + M₅ = \frac{cₐh^2bφh}{48} + \frac{5cₐh^2bφ7h}{48} = \frac{cₐh^3b}{24}. \]  

(2)

where the last term is critical as an exact value in connection with further comparison.

Vibration Equations. The conditions of dynamic equilibrium (D’Alambert)

\[ M\dddot{z} + R₁ = V(t); \]  

(3)
\[ M\dddot{x} + R_2 + (R_4 - R_5) = P(t); \]  
\[ \theta_0 \dddot{\varphi} - M_2 + 2M_5 + (M_4 + M_3) = m(t), \]

with whose use the final equations of the oscillations of a deeply embedded fixed foundation can be obtained after substitution of the elastic reactions and their moments

\[ M\dddot{x} + c_2 \alpha b z = V(t); \]
\[ M\dddot{x} + \frac{c_y h b}{2} - \frac{c_a h^2 b}{12} - \varphi = P(t); \]
\[ \theta_0 \dddot{\varphi} - \frac{c_y h^2 b}{12} - \frac{(c_a s b^2 + \frac{c_a h^3 b}{24})}{ \varphi = m(t).} \]

The underscored term of the equation is new.

The geometric sense of the coefficients in Eqs. (6), (7), and (8) can be readily noted (see Fig. 1), if the lower surface is examined first, and then the lateral side of the foundation. On the one hand, obviously, \( ab \) and \( a^3 b/12 \), are, respectively: the area and moment of inertia of the lower surface of the foundation. On the other hand, however, \( h b/2, h^2 b/12, \) and \( h^3 b/24 \) are, respectively, halves of: the area, resisting moment, and moment of inertia of the lateral side of the foundation, as Glushkov should have been presented, but did not in [1, 2].

Below, the errors in [1, 2] are denoted by an asterisk (*) positioned as a superscript. Two parabolas \( f_1 \) and \( f_2^* \) (see Fig. 2) are used instead of the single parabola \( f_1 \) in [1, 2]. This is erroneous, since the lateral pressure of the soil varies uniformly over the height of the foundation along the parabola \( f_1 \).

The parabola \( f_2^* \) is used to determine the arm \( r_s^* = 0.375h \) in lieu of the single parabola \( f_1 \) in [1, 2] from which it follows that \( r_s = 0.35h \).

Use of \( r_s^* \) yields a relative error \( \delta_1 = 7\% \).