We consider a model problem for the Stokes equations in the half-plane $\mathbb{R}_+^2$ $(x_2 > 0)$ with different boundary conditions on the semi-axes $(x_2 = 0, x_1 < 0)$ and $(x_2 = 0, x_1 > 0)$, which plays an important role in the studies of some free boundary problems, such as problem of filling or drying a capillary. The proof of the solvability of the problem in weighted Sobolev and Hölder spaces is presented, and estimates for the solution as well as the asymptotic formula for the solution in the vicinity of the singular point $x = 0$ are obtained. The proof is based on an explicit formula for the solution in terms of its Mellin transform, which makes it possible to obtain the estimates uniform with respect to one of the parameters of the problem (in the problem of filling a capillary it is proportional to the velocity of filling). Bibliography: 9 titles.
It is also necessary to consider the spaces of traces of elements of $H^m_{\mu}(\mathbb{R}^2_+)$, $m > 0$, on $\mathbb{R}^-$ and $\mathbb{R}^+$. These spaces are denoted by $H^m_{\mu-\frac{1}{2}}(\mathbb{R}^\pm)$, and the corresponding norms are given by

$$\|u\|_{H^m_{\mu-\frac{1}{2}}(\mathbb{R}^\pm)}^2 = \sum_{0 \leq j \leq m-1} \int_{\mathbb{R}^\pm} \left( \frac{\partial^j u}{\partial x_1^j} \right)^2 x_1^{2\mu-2(m-j)+1} dx_1 + \int_{\mathbb{R}^\pm} x_1^{2\mu} dx_1 \int_{|y_1-x_1|<|x_1|} \left( \frac{d^{m-1}u(x_1)}{dx_1^{m-1}} - \frac{d^{m-1}u(y_1)}{dx_1^{m-1}} \right)^2 \frac{dx_1 dy_1}{|x_1-y_1|^2}.$$ 

By the same symbols ($H^m_{\mu}$, $C^s$, $H^m_{\mu-\frac{1}{2}}$) we denote the spaces of vector fields with components belonging to the corresponding weighted spaces.

The following theorem on the solvability of problem (1) is a special case of the general results of [3-5].

**Theorem 1.** (i) For arbitrary $\tilde{f} \in H^m_{\mu}(\mathbb{R}^2_+)$, $r \in H^{m+1}_{\mu}(\mathbb{R}^2_+)$, $\tilde{d} \in H^{m+\frac{3}{2}}_{\mu}(\mathbb{R}^-)$, and $b, d \in H^{m+\frac{1}{2}}_{\mu}(\mathbb{R}^+)$, problem (1) has a unique solution $\tilde{v} \in H^{m+2}_{\mu}(\mathbb{R}^2_+)$, $p \in H^{m+1}_{\mu}(\mathbb{R}^2_+)$, provided that the number $z = m + 1 - \mu$ is not a root of the equations

$$\cos z\pi = 0,$$  
$$\tan z\pi + 2k\nu = 0.$$ 

The solution satisfies the inequality

$$\|\tilde{v}\|_{H^{m+2}_{\mu}(\mathbb{R}^2_+)} \leq C_1 (\|\tilde{f}\|_{H^m_{\mu}(\mathbb{R}^2_+)} + \|r\|_{H^{m+1}_{\mu}(\mathbb{R}^2_+)} + \|\tilde{d}\|_{H^{m+\frac{3}{2}}_{\mu}(\mathbb{R}^-)} + \|b\|_{H^{m+\frac{1}{2}}_{\mu}(\mathbb{R}^+)} + \|d\|_{H^{m+\frac{1}{2}}_{\mu}(\mathbb{R}^+)}).$$  

(ii) If $\tilde{f} \in H^m_{\mu}(\mathbb{R}^2_+) \cap H^m_{\mu}(\mathbb{R}^2_+)$, $r \in H^{m+1}_{\mu}(\mathbb{R}^2_+) \cap H^{m+1}_{\mu}(\mathbb{R}^2_+)$, $\tilde{d} \in H^{m+\frac{3}{2}}_{\mu}(\mathbb{R}^-) \cap H^{m+\frac{3}{2}}_{\mu}(\mathbb{R}^-)$, and $b, d \in H^{m+\frac{1}{2}}_{\mu}(\mathbb{R}^+) \cap H^{m+\frac{1}{2}}_{\mu}(\mathbb{R}^+)$, neither $m + 1 - \mu$, nor $m' + 1 - \mu'$ is a root of (2), (3), and $m + 1 - \mu < m' + 1 - \mu'$, then the solutions $(\tilde{v}, p) \in H^{m+2}_{\mu}(\mathbb{R}^2_+) \times H^{m+1}_{\mu}(\mathbb{R}^2_+)$ and $(\tilde{v}', p') \in H^{m'+2}_{\mu}(\mathbb{R}^2_+) \times H^{m'+1}_{\mu}(\mathbb{R}^2_+)$ of problem (1) are related to each other by

$$\tilde{v} - \tilde{v}' = \sum_j a_j \tilde{v}_j, \quad p - p' = \sum_j a_j p_j.$$  

Here $\tilde{v}_j$ and $p_j$ are solutions of the homogeneous problem (1) of the form

$$\tilde{v}_j(x) = \rho^{z_j} \tilde{V}(\varphi), \quad p_j(x) = \rho^{z_j-1} P_j(\varphi),$$  

where $(\rho, \varphi)$ are the standard polar coordinates in $\mathbb{R}^2$, $z_j$ are roots of (2), (3), and the summation in (5) is extended to all $z_j$ in the interval $(m + 1 - \mu, m' + 1 - \mu').$

A similar theorem holds for weighted Hölder spaces.

**Theorem 2.** (i) For arbitrary $\tilde{f} \in C^t_{s-2}(\mathbb{R}^2_+)$, $r \in C^t_{s-1}(\mathbb{R}^2_+)$, $\tilde{d} \in C^{t+1}_{s-1}(\mathbb{R}^-)$, and $b, d \in C^{t+1}_{s-1}(\mathbb{R}^+)$, problem (1) has a unique solution, provided that $s$ is not a root of (2), (3). The solution satisfies the inequality

$$|\tilde{v}|_{C^{t+2}_{s,1}(\mathbb{R}^2_+)} + |p|_{C^{t+1}_{s,1}(\mathbb{R}^2_+)} \leq C_2 (|\tilde{f}|_{C^t_{s-2}(\mathbb{R}^2_+)} + |r|_{C^t_{s-1}(\mathbb{R}^2_+)} + |\tilde{d}|_{C^{t+1}_{s-1}(\mathbb{R}^-)} + |b|_{C^{t+1}_{s-1}(\mathbb{R}^+)} + |d|_{C^{t+1}_{s-1}(\mathbb{R}^+)}).$$  

(ii) If $\tilde{f} \in C^t_{s-2}(\mathbb{R}^2_+) \cap C^t_{s-2}(\mathbb{R}^2_+)$, $r \in C^t_{s-1}(\mathbb{R}^2_+) \cap C^t_{s-1}(\mathbb{R}^2_+)$, $\tilde{d} \in C^{t+1}_{s-1}(\mathbb{R}^-) \cap C^{t+1}_{s-1}(\mathbb{R}^-)$, $b, d \in C^{t+1}_{s-1}(\mathbb{R}^+) \cap C^{t+1}_{s-1}(\mathbb{R}^+)$, $s$ and $s'$ are not roots of (2), (3), and $s < s'$, then the solutions $(\tilde{v}, p) \in C^{t+2}_{s,1}(\mathbb{R}^2_+) \times C^{t+1}_{s,1}(\mathbb{R}^2_+)$