EQUATIONS OF GROWTH OF FATIGUE CRACKS IN INHOMOGENEOUS PLATES

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We propose a computational model for the determination of the kinetics of propagation of fatigue cracks in plates whose material has inhomogeneous mechanical characteristics and fatigue fracture resistance. We also take into account the history of plastic deformation of the plate. The model is based on the energy criterion of fatigue fracture of materials. We deduce a system of differential equations describing the kinetics of fatigue cracks in inhomogeneous plates and present the solutions of two particular problems obtained by using the proposed model.

The mechanical characteristics of materials are, as a rule, inhomogeneous. At the same time, the applicability of the existing analytic models of fatigue fracture [1, 2] to inhomogeneous materials seems to be doubtful. Moreover, the laws of propagation of fatigue cracks in inhomogeneous materials and, in particular, in welded elements, are studied quite poorly [3–5]. For this reason, in the present work, we make an attempt to formulate the kinetic equations of growth of fatigue cracks in plates with inhomogeneous fatigue fracture resistance. We also take into account the history plastic deformation of the material. In our model, we use the energy criterion of fatigue fracture based on the following hypothesis: In the process of growth of a fatigue crack, the intensity of dissipation of plastic strain energy per unit area of the freshly formed surface is a constant of the material, loading mode, and temperature [6–8].

Consider an elastoplastic plate weakened by a crack and subjected to cyclic tension. Assume that the increment of crack length attained for $\Delta N$ loading cycles is equal to $\Delta a$ (Fig. 1) and that a plastic zone of length $l_{pf} (\Delta a > l_{pf})$ is formed near the crack tip. It is known [9] that $l_{pf}$ is smaller than the length of the static plastic zone $l_p$ and depends on the load ratio $R$:

$$l_{pf} = \frac{(1 - R)^2}{4} l_p.$$

To construct equations of crack growth, we use the energy criterion of fracture according to which, for any material, there exists a critical level of energy $W_p$ required for an elementary event of fracture (fracture energy of the material). Thus, this criterion states that, in order that the increment of length of a fatigue crack for $\Delta N$ loading cycles be equal to $\Delta a$, the total amount of energy dissipated in the process of plastic deformation of the material $W$ must be equal to the fracture energy of the material $W_p$, i.e.,

$$W = W_p,$$

or

$$W_0 + W_s + \Delta N W_c = W_p,$$  \hspace{1cm} (1)

where $W_s$ is the energy acquired under static loading up to the maximum load in a cycle, $W_c$ is the amount of plastic strain energy dissipated for a loading cycle, and $W_0$ is the amount of plastic strain energy dissipated prior to cyclic loading. Since, for a single event of fracture, the crack tip opening displacement is maximum ($\delta_{\text{max}}$) at each point of the segment $\Delta a$, the energy $W_s$ is given by the formula

where \( \sigma_{0f} \) is the average level of stresses in the prefracture zone according to the \( \delta_c \)-model [10] with regard for the cyclic hardening and softening of the material [11]. The tensile stress-strain diagram of the material is approximated by a diagram typical of a perfectly elastoplastic material (Fig. 2) and we assume that

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\sigma_{0f} = \frac{\sigma_y + \sigma_u}{2},
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