PROPAGATION OF PLASTICITY BANDS FROM THE TIP OF A SEMIINFINITE CRACK OF MIXED TYPE

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In the framework of the model of plasticity bands, we analyze an elastoplastic fracture-mechanics problem of propagating plasticity bands near the tip of a semiinfinite crack of mixed type. It is assumed that, under the conditions of plane stressed state, plastic strains near the tip of a semiinfinite crack are localized along three plasticity bands \((L_1, L_2, L_3)\) and, under the conditions of plane deformation, they are localized along two bands \((L_1, L_2)\). One of these bands \((L_3)\) is simulated by a line of discontinuity of normal and tangential displacements. The remaining two bands \((L_1, L_2)\) are simulated by the lines of discontinuity of tangential displacements. Their lengths and orientations are determined in the process of numerical solution of the problem by the method of singular integral equations. We also present the values of crack tip opening displacements.

In the vicinity of crack tips, materials always undergo plastic deformation. To analyze plasticity zones, it is customary to use the model of plasticity bands regarded as surfaces of discontinuities of displacements satisfying the conditions of plasticity. Beyond these surfaces, the body is supposed to be elastic [1, 2]. However, within the framework of this model, it is mathematically difficult to describe the onset of plastic deformation (small-scale yield) for a crack of finite length [3–7]. In the present work, by using the method of singular integral equations, we develop a general approach to the solution of the problem of propagation of slip bands near a semiinfinite crack of mixed type (mode I + II) under the conditions of plane stressed state or plane deformation. We determine crack tip opening displacements and the lengths of plasticity bands under general loading. For some cases, we also present approximate formulas in the explicit form. The solution of this problem can be regarded as the asymptotics of the corresponding solution of the problem of propagation of plasticity zones near the tip of a crack of finite length under the conditions of small-scale yield.

Plastic deformation near the crack tip in the case of small-scale yield was studied by using various approaches [8–11]. Most often, these investigations deal with tensile cracks for which the plasticity zone is symmetric about the crack line [8, 9]. The problems of propagation of the plasticity zone near the tip of a semiinfinite crack of mixed type in a homogeneous body [10] and along the interface of rigid and elastic materials [11] were studied only under the assumption that the crack tip serves as the origin of a single slip band (simulated by a jump of tangential displacements). In [5, 6], the parameters of nonlinear fracture mechanics are evaluated for a crack of mixed type.

Plane Stressed State

Consider a semiinfinite crack \(L_0\) with plasticity bands in an infinite plate. We assume that three bands \((L_1, L_2, L_3)\) of different lengths and orientations originate from the tip of this crack (Fig. 1). The crack \(L_0\) is directed along the \(Ox\)-axis \((x \geq 0)\). The contours \(L_k, k = 1, 3\), are related to local coordinates \(x_kO_ky_k\) whose origins are located at the points \(O_k\) (at the centers of the cuts \(L_k, k = 1, 3\)) so that the \(O_kx_k\)-axes coincide with the lines of the cuts. The contours \(L_k, k = 1, 3\), make angles \(\pi - \alpha_k\) with the \(Ox\)-axis. Plastic strains in the band \(L_3\) responsible for the effect of local thinning of the plate are simulated by the jumps of tangential and normal displacements. The band propagates in the direction of maximum tensile stresses. In the bands \((L_1, L_2)\) propagating in the plane of maximum tangential stresses [2], plastic strains are simulated by the jump of tangential displacements.
Assume that the lips of the crack $L_0$ are free of stresses and the stressed state in the plate without plasticity bands is characterized by the stress intensity factors $K^0_I$ and $K^0_{II}$. In this case, the expressions for the complex potentials of stresses formed in the region under consideration take the form [12]

$$
\Phi_0(z) = \frac{K^0_I - iK^0_{II}}{2\sqrt{2\pi z}} \quad \text{and} \quad \Psi_0(z) = \frac{K^0_I + 3iK^0_{II}}{4\sqrt{2\pi z}},
$$

where $\sqrt{z} = r^{1/2}e^{-i\theta/2}$, $0 \leq \theta \leq 2\pi$, and $r, \theta$ are polar coordinates whose pole is located at the tip of the crack $L_0$ and the axis coincides with $Ox$. We also assume that the material of the plate is perfectly elastoplastic and the Tresca–Saint-Venant plasticity condition is satisfied along the slip lines. In this case, we have the following limiting expressions for stresses on the lips of the cuts simulating the plasticity bands [2]:

$$
N^+ + iT^+ = \sigma_y = 2\tau_y, \quad t_n \in L_n, \quad n = 3,
$$

$$
N^- = N_n, \quad v^+ = v_n^-, \quad T^\pm = (-1)^{n+1}\tau_y, \quad n = 1, 2,
$$

where $N_n$ and $T_n$ are, respectively, the tangential and normal components of stresses, $v_n$ is the projection of the displacement vector onto the $O_n\alpha_n$-axis (all quantities are given in local coordinates $x_nO_n\alpha_n$ and the superscripts “+” and “−” correspond to the upper and lower lips of the cut, respectively), $t_n = x_n + iy_n$, and $\sigma_y (\tau_y)$ is the tensile (shear) yield strength of the material. Therefore, the problem of propagation of plasticity bands is reduced to a problem of the two-dimensional theory of elasticity for a body with branched cut whose branches are subjected to the action of forces (2) but the angles of orientation $\alpha_n, n = \overline{1,3}$, and lengths $l_n, n = \overline{1,3}$, are unknown. Under the assumption that stresses in the elastoplastic body are bounded, the stress intensity factors obtained as a result of the solution of the corresponding elastic problem are equal to zero at the tips of the branches $L_n, n = \overline{1,3}$, namely,

$$
K_{II}(l_n, \alpha_n) = 0, \quad t_n \in L_n, \quad n = \overline{1,3},
$$

$$
K_I(l_3, \alpha_3) = 0, \quad t_3 \in L_3.
$$