PROBLEMS OF THERMODIFFUSION OF DEFORMABLE SOLIDS

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This is a brief survey of the problems and characteristic features of the thermodiffusion of solid deformable bodies. We show the subject in its historical development commencing from the fundamental Fourier's and Fick's ideas. We emphasize the importance of the works devoted to thermal diffusion (referring to fluids) and the development of the thermodiffusion of deformable solid bodies in the 60s first discussed by Ya. Pidstryhach and his colleagues and then by W. Nowacki and many others. We point out the early papers by C. Truesdell and other mathematicians who contributed to make the theory of thermodiffusion rigorous. Likewise, the papers in the general theory of coupled fields in elastic and inelastic solid bodies are cited.

When speaking about thermodiffusion, one must first mention the contribution made by two famous scientists, namely, by François Marie Charles Fourier, who successfully worked in many fields of mathematics, and A. Fick. As early as in 1822, Fourier derived equations of thermal conductivity [1]. His law of heat conduction states that the components of a heat flow are linear functions of the components of the temperature gradient. More than thirty years later, Fick followed Fourier's way of thinking to derive equations of diffusion conductivity. This result was published in 1855 [3]. In Fick's law, it is assumed that the diffusion flows are linear functions of the components of the gradient of mass concentration. In both cases, the derived equations can be reduced to parabolic partial differential equations. This happens when the relevant material coefficients are assumed to be constant. Clearly, at the times of Fourier and Fick, the phenomena of heat conduction and conduction of mass diffusion were treated independently and separately, and no coupling of the fields of temperature and diffusion was taken into account. From the standpoint of contemporary rational mechanics, these two partial differential equations are particular cases of balance equations (for the processes of heat and mass transfer, respectively; see, e.g., [19, 48, 112]. If we consider the equations of thermal conductivity in Fourier's form and diffusion conductivity in Fick's form, then we get two independent systems of equations. The idea of coupling the fields of temperature and diffusion came much later. L. Dufour [5, 6] was the first who discovered that the gradient of diffusing substance also induces a certain flow of energy. This phenomenon is now called Dufour's effect. On the other hand, the effect of generation of a certain flow of diffusing substance by a heat flux is called Soret's effect, after the first scientist who considered problems of this sort (compare [7, 8] with [72]. Although it seems that the indicated two effects, i.e., Dufour's effect and Soret's effect, should be symmetric. In fact, there exists a fundamental difference between the phenomena of transport of energy and transport of mass. The flow of heat tends to equalize temperatures and, hence, the system approaches the state of thermodynamic equilibrium. On the contrary, the state formed in the system as a result of the flow of mass may be remote from equilibrium. Both these effects were investigated in liquids and gases.

In 1931, L. Onsager established his "reciprocal relations" connecting the coefficients in the linear phenomenological laws used to describe the irreversible thermodynamical processes. In nonequilibrium thermodynamics, the central role is played by the so-called entropy balance equation. It expresses the fact that the entropy changes with time for two reasons. First, because of the entropy flow into an element of the volume and, second, due to the existence of the sources of entropy produced by irreversible thermodynamical processes. The Onsager–Casimir reciprocity theorem enables one to deduce numerous relations reducing the number of independent quantities and relating the physical effects to each other. The notions of thermodynamic forces and fluxes were introduced. Onsager assumed that, in the first approximation, the fluxes are linear functions of the thermodynamic forces. Not only the Fourier law of heat conduction and the Fick law of diffusion belong to this category but also, e.g., the Ohm law of electric conductivity, the laws of chemical reactions, and the cross effects. In Onsager's theory, an important role is played by Curie's principle ([9, 21]). It says that the existence of spatial symmetry properties may simplify the form...
of phenomenological equations in such a way that the Cartesian components of the fluxes become independent of all Cartesian components of the thermodynamic forces. In addition, a thermodynamic force of higher tensorial symmetry cannot be generated by a cause of lower symmetry. Thus, e.g., a scalar cause cannot generate an effect of vectorial character or an effect conjugated with vectorial.

In thermodynamics and chemistry, the researchers are rather interested in the processes running in liquids or/and gases (i.e., in fluids). The theory deals with binary, ternary, and multicomponent systems. In case of heat and substance fluxes, it is called “thermal diffusion” unlike “thermodiffusion” in the case of solid bodies. The characteristic parameters of the cross effects, i.e., of Soret’s and Dufour’s effects are discussed and the corresponding coefficients (the heat conductivity coefficient, the Dufour coefficient, and the coefficient of thermal diffusion or the Soret coefficient and the diffusion coefficient) are theoretically established and experimentally measured. These problems were discussed in the 30s (cf., e.g., [21]).

Many particular problems in the fields of thermoelasticity and thermoplasticity of solid bodies have been solved having in mind practical purposes, including military applications. The same refers to the problems of drying materials as, e.g., wood, and the processes of heat processing of metals. Of great importance are the problems of buckling of structures in wet environments and buckling of railway tracks at elevated temperatures. On the other hand, solids with cracks, inclusions, and inhomogeneities are very sensitive to temperature variations and moisture in the ambient medium.

The development of high technologies in the years before, during, and after the second world war pronouncedly affected the investigations in which the fields of temperature and diffusion in solids cannot be neglected. At elevated and low temperatures, the processes of heat and mass transfer play the decisive role in many problems of satellites, returning space vehicles, and landing on water or land. It is not necessary to prove how important is to find the stresses generated in solids by frictional heating.

The problems of thermoelasticity, as a developed theory, were discussed in many investigations, works, and monographs. Here, we mention the monographs [15, 18, 20, 34, 44, 86] mainly devoted to the analysis of the linear problems of thermoelasticity. At the same time, diffusion in solids was also studied by physicists (Z. Adda and J. Philibert [32], A. Alfrey et al. [33], J. Crank [57], M. Jacobs [38], W. Jost [12]), metallurgists [25, 53], and experts in the field of polymers [11, 33, 35, 43].

The fundamentals of diffusion and heat transfer were presented from the standpoint of contemporary thermodynamics, together with the discussion of the principles of constitutive equations, in a number of papers by American mathematicians (see C. Truesdell [19, 48] and the list of references in [48]). The fundamentals of diffusion and thermal diffusion in mixtures were studied by I. Müller [39, 42]. In this connection, it is interesting to mention an early paper by J. C. Maxwell [4].

The first theoretical works in the field of thermodiffusion of elastic deformable solid bodies belong to Ya. Pistryhach who deduced fundamental equations of linear thermodiffusion in 1961 [16]. With a group of his colleagues (V. Pavlyna, P. Shevchuk, N. Shvets, and Ya. Dasyuk [65]), he published a series of works (see [16, 17, 22–24, 27–31, 40, 45]). W. Nowacki, started from the linear thermoelasticity of elastic solids ([18, 20, 34]), continued, and later developed the theory of thermodiffusion of elastic solids (see, e.g., [47, 52, 58, 61]). For more than two decades, the problems of thermodiffusion of solid bodies were popular among Polish authors (cf. the works by J. Madejski [26], M. Buda [67, 70], M. Dryja [68], K. Grysa and R. Szczepański [72], T. Hoffman [95], Z. Kończak, and Z. Sobczyńska-Kończak [73, 74], J. Kubik [83, 87, 96], J. Stefaniak and J. Jankowski [79], and J. Wyrwal [88, 100]. As a result of international collaboration, we are able to cite the papers written by Yugoslavian researchers who spent long periods of time on scholarships in Warsaw (R. Cukić [56], and N. Naerlović-Veljković [54]) and Ukrainian scientists R. Mokryk [78] and Yu. Pyryev [119]. Since the scientific interests of Ukrainian and Polish researchers coincided, joint conferences were organized alternately in Warsaw and Kyiv on the initiative of W. Nowacki. The reports on these conferences can be found in the Journal of Theoretical and Applied Mechanics (Warsaw). The first conference devoted to nonclassical problems of the theory of elasticity took place in Warsaw (October 26–29, 1970) and was organized by the Committee for Mechanics and Physics of Continuous Media of the Polish Academy of Sciences. The second conference was organized by the Institute of Mechanics of the Ukrainian Academy of Sciences in October 1971. The last (sixth) joint conference was organized, to my knowledge, in 1979. Later, the well-known events, far from scientific activities, interrupted the development of collaboration. Only in 1995, this time on the