THERMOELASTOPLASTIC EQUILIBRIUM OF A CIRCULAR ECCENTRIC TUBE

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We formulate the problem of thermoelastic and thermoelastoplastic equilibrium of a circular eccentric tube and obtain its approximate solution under the assumption that the mechanical and thermal characteristics of the material are independent of temperature.

There are several works (see, e.g., [1–3]) devoted to the analysis of the problems of symmetric thermoelastic and thermoelastoplastic equilibrium states of a circular cylindrical tube. For eccentric tubes, the elastoplastic problem was studied earlier (see, e.g., [4]).

We consider thermoelastic and thermoelastoplastic problems for a circular eccentric tube operating in a nonuniform temperature field induced by a heat flux on the inner surface of the tube and subjected to the action of internal and external pressure forces.

Statement of the Problem

Consider a long eccentric circular tube with outer radius $b$ and inner radius $a$ (see Fig. 1). The axes of symmetry of the surfaces of the tube are located at a distance $e$ from each other and this distance is small as compared with the radii of the tube. The lateral surfaces of the tube are subjected to the action of normal compressive stresses $p_a$ and $p_b$. At the same time, tangential stresses are equal to zero on these surfaces. In addition, on the inner surface of the tube, we specify a heat flux of constant intensity and assume that the process of heat exchange between the outer surface of the tube and the ambient medium (whose temperature is equal to zero) obeys the Newton law with a constant coefficient of heat transfer. It is also supposed that the mechanical and thermal characteristics are independent of temperature, the material is perfectly elastoplastic, incompressible in the plastic zone, and satisfies the Tresca–Saint-Venant plasticity condition. In addition, we assume that the axial strains in the tube are constant, their levels in the elastic and plastic zones coincide, and the tube is subjected to the action of an axial force.

It is necessary to determine the temperature field, stresses and displacements in the elastic and plastic zones, the boundary between the elastic and plastic zones, and the level of axial strains in the tube.

The problem is posed as two-dimensional. To solve it, we use a polar coordinate system $(r, \varphi)$ whose pole is located at the center of the inner circle of the cross section of the tube. First, we find the temperature field in the tube and its thermoelastic equilibrium state. Then we determine its thermoelastoplastic state.

Thermoelastic Problem

From the mathematical point of view, the problem is reduced to the solution of the stationary heat conduction equation

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \varphi^2} = 0$$

with the following boundary conditions:
Fig. 1. Schematic diagram of the problem; \( \Gamma \) is the boundary of the elastic and plastic zones.

\[
\begin{align*}
\tau &= \tau_0 = \alpha F - f, \\
\tau &= \tau_b = -\gamma t.
\end{align*}
\]

It is also necessary to find the solution of the biharmonic equation for the stress function \( \Phi(r, \varphi) \), i.e.,

\[
\Delta^2 \Phi(r, \varphi) = 0,
\]

where

\[
\Delta = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2}
\]

is the Laplace operator.

The stresses can be determined by using the following formulas:

\[
\begin{align*}
\sigma_r &= -\frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2}, \\
\tau_{\varphi \varphi} &= -\frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2}, \\
\sigma_\varphi &= \frac{\partial^2 \Phi}{\partial r^2}, \\
\sigma_z &= \nu (\sigma_r + \sigma_\varphi) + E (e - \alpha t).
\end{align*}
\]

The mechanical boundary conditions have the form

\[
\begin{align*}
r &= a: & \sigma_r &= -p_o, & \tau_{\varphi \varphi} &= 0, \\
r &= b: & \tau_\varphi &= 0.
\end{align*}
\]