A SYSTEM OF BOUNDARY INTEGRAL EQUATIONS
FOR ORTHOTROPIC SHELLS OF POSITIVE CURVATURE
WITH SLITS AND HOLES

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We propose a method of constructing a system of boundary integral equations for the problem of the stress state of an orthotropic shell with slits and holes. Using the theory of distributions and the two-dimensional Fourier transform, we reduce the problem to a system of boundary integral equations. In the solution obtained the kernels of the system of integral equations do not contain the direction cosines of the unit outward normal vector explicitly. There are no extra-integral terms. The matrix of the kernels is symmetric. The kernels are regular or have a logarithmic singularity. Two figures. Bibliography: 6 titles.

Khizhnyak, Shevchenko, and the author [1; 2] have used the theory of distributions and the two-dimensional Fourier transform to construct boundary integral representations of the theory of shallow orthotropic shells with straight-line slits. By choosing the unknown functions as a combination of the displacement jumps and angles of revolution and their derivatives and integrals, Shevchenko and the author [3] obtained in explicit form the kernels of the integral equations for an orthotropic shell with slits and holes of arbitrary configuration. In the present paper we propose a new method of constructing a system of boundary integral equations for the problem of the stress state of an orthotropic shell with slits and holes. When this method is used, the kernels of the system are regular or have a logarithmic singularity, and they do not contain explicitly the direction cosines of the outward unit normal vector. The matrix of the kernels is symmetric, and there are no extra-integral terms.

Consider an orthotropic shell of positive Gaussian curvature and constant thickness \( h \) weakened by a system of curvilinear slits and holes with piecewise-smooth boundary curves. We choose a system of orthogonal coordinates \( x, y \) with axes along the directions of elastic and geometric symmetry of the shell. We denote the smooth portions of the boundary curves of the slits and holes by \( L_m \) (\( m = 1, N \), see Fig. 1). We shall assume that in the coordinate system chosen the equations of \( L_m \) have the form

\[
x = \alpha_m(s), \quad y = \beta_m(s), \quad s \in [-l_m, l_m],
\]

where \( \alpha_m(s) \) and \( \beta_m(s) \) are continuously differentiable functions, and \( l_m \) is the half-length of the curve \( L_m \). We assume that the boundaries of the slits and holes are free of external loads, and that the edges of the slits do not touch one another.

We represent the stress state of this shell as the sum of the stress states in the shell without slits and holes under the given external load, which we assume known (it will be denoted below by quantities with an asterisk), and the perturbed stress state caused by the presence of the slits and holes.

The boundary conditions for the perturbed field have the form

\[
T_{n_m} = -T_{n_m}^*; \quad S_{n_m} = -S_{n_m}^*; \quad M_{n_m} = -M_{n_m}^*; \quad Q_{n_m} = -Q_{n_m}^* \quad \text{on} \quad L_m \quad (m = 1, N).
\]

Here \( T_{n_m} \) and \( S_{n_m} \) are the membrane forces, \( M_{n_m} \) is the bending moment, and \( Q_{n_m} \) is the generalized shear force.

We shall solve the boundary-value problem (2) using the equations of the theory of orthotropic shells with large index of variability, which coincide with the equations of shallow shells [4]. To obtain the kernels of the integral equations in explicit form we shall combine the boundary conditions and their integrals.

Consider an arbitrary curve \( L \). We denote by \( L_s \) the part of \( L \) with an endpoint at the variable point \( s \) (Fig. 2). For the principal vector \( \vec{P} \) of forces acting along \( L_s \) and their principal moment \( \vec{M}^{(s)} \) relative to the point \( s \) we have

\[
P_x(s) = \int_{L_s} T_n \, dy + S_{nT} \, dx; \quad P_y(s) = \int_{L_s} S_{nT} \, dy - T_n \, dx; \quad P_z(s) = \int_{L_s} N_s \, ds - k_2 P_y \, dy - k_1 P_x \, dx;
\]

\[
M_x^{(s)}(s) = - \int_{L_s} \left( \int_{L_t} Q_n \, dt - \int_{L_t} k_2 P_y \, dy + k_1 P_x \, dx \right) \, dy - (M_n - k_1 M_z^{(s)}) \, ds;
\]

\[
M_y^{(s)}(s) = \int_{L_s} \left( \int_{L_t} Q_n \, dt - \int_{L_t} k_2 P_y \, dy + k_1 P_x \, dx \right) \, dx + (M_n - k_2 M_z^{(s)}) \, ds;
\]

\[
M_z^{(s)}(s) = \int_{L_s} P_x \, dy - P_y \, dx,
\]

where \( N_n \) is the shear force, and \( k_1 \) and \( k_2 \) are the principal curvatures of the shell.

Using the theory of distributions and the two-dimensional Fourier transform, along with the integral representations of the displacements [1] and the fundamental solutions of the equations of statics of shallow orthotropic shells [5], we obtain the following system of boundary integral equations:

\[
\sum_{p=1}^{N} \sum_{j=1}^{5} K_{ij}((\alpha_m(t) - \alpha_p(s))/\chi, \beta_m(t) - \beta_p(s))\psi_{jp}(s) \, ds = \pi^2 F_i^*(t),
\]

\[
t \in (-l_m, l_m), \quad m = 1, N, \quad i = 1, 5.
\]

Here

\[
\psi_{1p} = \frac{Eh(1 - \mu)}{a} \left( \frac{\partial[u]_{L_p}}{\partial t} - n_2 \frac{\lambda}{R_2} [w]_{L_p} \right),
\]

\[
\psi_{2p} = \frac{Eh(1 - \mu)}{a} \left( \frac{\partial[v]_{L_p}}{\partial t} + n_1 \frac{[w]_{L_p}}{R_2} \right),
\]

\[
\psi_{3p} = \frac{Eh(1 - \mu)}{a} \frac{1}{c^2 R_2} \frac{\partial[\theta_1]_{L_p}}{\partial t},
\]

\[
\psi_{4p} = \frac{Eh(1 - \mu)}{a} \frac{1}{c^2 R_2} \frac{\partial[\theta_2]_{L_p}}{\partial t},
\]

\[
\psi_{5p} = \frac{Eh(1 - \mu)}{a} \frac{1}{c^2 R_2} (n_1 [\theta_1]_{L_p} + \lambda n_2 [\theta_2]_{L_p});
\]

\[
F_1(s) = -P_x(s) + c_1, \quad F_2(s) = -P_y(s) + c_2,
\]

\[
F_3(s) = c^2 R_2 \left( M_y^{(s)}(s) - \frac{1}{R_2} (c_1 (\lambda x^2 - y^2)/2 + 2c_2 xy - c_3 y) + c_6 x + c_3 \right),
\]

\[
F_4(s) = c^2 R_2 \left( M_x^{(s)}(s) - \frac{1}{R_2} (c_2 (\lambda x^2 - y^2)/2 - 2c_1 \lambda xy - \lambda c_3 y) - c_6 y + c_4 \right),
\]

\[
F_5(s) = c (M_z^{(s)}(s) - yc_1 + xc_2 + c_5);
\]

\[
\chi^2 = \sqrt{E_1/E_2}; \quad c^2 = \sqrt{12(1 - \nu^2)/R_3 b}; \quad \lambda = R_2/R_1; \quad \nu = \sqrt{\nu_1 \nu_2}; \quad [f]_{L_p} = f^+ - f^- \quad \text{is the jump of the function } f \text{ when crossing the curve } L_p \text{ from the direction of the outward normal; } u, v, w \text{ are the components of the displacement vector; } n_1 \text{ and } n_2 \text{ are the direction cosines of the outward unit normal vector to } L_p; \quad \theta_1 \text{ and}
\]

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