ANISOTROPOVISCOUS MODELS OF THE PHENOMENON OF REDUCTION OF HYDRODYNAMIC RESISTANCE

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We construct anisotropoviscous two- and three-layer models of the effect of reduction of hydrodynamic resistance through the addition of polymers. The three-layer model can be used to compute the hydrodynamic resistance of turbulent flows with polymer additives in the entire region of variation of the Reynolds numbers, and the two-dimensional model can be used with large Reynolds numbers. Two figures. Bibliography: 3 titles.

The second author [1] has constructed a semiempirical theory of the effect of reduction of hydrodynamic resistance through the addition of polymers in the context of a structural approach. The basis of the theory is the mechanism of resonance absorption of turbulent energy of macromolecules of polymers. Here at each point of the flow a symmetric tensor of rank two $\mu_{ij}$ is prescribed, whose components have the dimension of dynamic viscosity. In the present article, by introducing the averaged two- and three-layer models of the flow of a fluid with polymer additives, we simplify the basic relations obtained in [1].

For a two-dimensional flow the longitudinal component $\mu_{11}$ (in the direction of flow) and the transverse component $\mu_{22}$ of the tensor, in terms of which the coefficient of anisotropy of dynamic viscosity $A = \frac{\mu_{22}}{\mu_{11}}$ is computed, are decisive. For such a flow [1]

$$A = 1 + f_1(c, M, \gamma) f_2(\omega_2, \omega_1, N, \xi),$$

where

$$f_1 = 0.188 \arctan(0.166cM^{0.85}), \quad f_2 = \frac{12}{\pi} \sum_{p=1}^{N} \frac{1}{p} \arctan \left( \frac{\omega_2/\omega_1 - 1}{\omega_2/\omega_1 + p^2} \right);$$

$c$ is the concentration; $M$ is the molecular mass of the polymer; $\gamma$ is a parameter that takes account of the distention of the polymer molecules; $\omega_2 = \omega_2(y)$ is the distribution of the frequencies of the Kolmogorov perturbations over a section of the turbulent flow under consideration; $\omega_1 = \omega_1(y)$ is the distribution of the analogous frequencies at the threshold Reynolds number; $N$ is the number of segments in the molecular chain; and $\xi$ is a parameter that characterizes the distribution of relaxation times in the spectrum of the polymer molecule.

Using relation (1), we can construct a nonlayer model of turbulence near a wall [1], which makes it possible to compute the profile of the average velocity and hydrodynamic resistance in turbulent flows with polymer additives, including the maximal (limiting) reduction of resistance. However, in order to obtain simpler relations in the applied computations it makes sense to construct averaged two- and three-layer models of the flow of fluids with polymer additives. For these models the average coefficient of anisotropy of the dynamic viscosity $\bar{A}$ can be introduced, and in the expression for the function $f_2$ one can pass to the ratio of the averaged frequencies $\bar{\omega}_2/\bar{\omega}_1$.

Consider the integral Kolmogorov turbulence scale $\bar{\eta} = \frac{1}{\alpha} \int_0^\alpha \eta(y) dy$ averaged over the section of the flow and the frequency $\bar{\omega} = k_1\nu/\bar{\eta}^2$ corresponding to it [2], where $\alpha$ is the width of the flow, and $k_1$ is a constant of the order of 1. For these quantities we have

$$\bar{\eta} = \int_0^1 \eta(y/\alpha) d(y/\alpha),$$

$$\bar{\omega} \equiv \nu/\bar{\eta}^2.$$
We can then write the function \( f_2 \) in the form

\[
f_2 = \frac{12}{\pi} \sum_{p=1}^{N} \frac{1}{p} \arctan \left( \frac{(\eta_1 / \eta_2)^2 - 1}{(\eta_1 / \eta_2)^2 + p^2} \right).
\]

(5)

Formula (3) contains the Kolmogorov turbulence scale \( \eta = \eta(y) \), whose distribution over a sector of the flow can be found using the relations for the dissipation of turbulent energy \( \varepsilon \), the coefficient of turbulent viscosity \( \nu_T \) and the kinetic energy of turbulence \( q^2 \) [1]. Using these relations, we can represent the integral Kolmogorov scale as

\[
\hat{\eta} = c \eta^{3/4} \alpha^{1/4} u_*^{-3/4} \int_0^1 \varphi(\gamma/\alpha, \alpha/\delta, \lambda) d(\gamma/\alpha),
\]

(6)

where \( c \) is a universal constant; \( \nu \) is the coefficient of kinematic viscosity of the solvent (water); \( u_* \) is the dynamic velocity; \( \varphi \) is a dimensionless function that characterizes the distribution of the Kolmogorov scale over a section of the flow; \( \delta = 5.6\nu / u_* \) is the thickness of the viscous layer; and \( \lambda \) is the coefficient of hydrodynamic resistance.

At large Reynolds numbers, when the effect of reduction of hydrodynamic resistance manifests itself in flows with polymer additives, the integrand \( \varphi \) changes insignificantly as the arguments \( \alpha/\delta \) and \( \lambda \) vary. In this case one can assume \( \varphi = \varphi(y/\alpha) \). Then for the ratio of the Kolmogorov scales we find

\[
n = \frac{\eta_1}{\eta_2} = \left( \frac{u_{*,2}}{u_{*,1}} \right)^{3/4}.
\]

(7)

Here \( u_{*,1} \) is the threshold value of the dynamic velocity, and \( u_{*,2} \) is the dynamic velocity of the flow in question.

Taking account of (7), we have for the averaged coefficient of anisotropy of dynamic viscosity

\[
A = 1 + 0.72 \arctan(0.166 cM^{0.85}) \sum_{p=1}^{N} \frac{1}{p} \arctan \left( \frac{n^{3/2} - 1}{p^2 + n^{3/2}} \right).
\]

(8)

The expression (8) can be used to construct two- and three-layer models for the reduction of hydrodynamic resistance in turbulent flows with polymer additives.

According to the schematization adopted in the theory of turbulence for the two-layer model [2] we shall assume that all the action of the polymer additives is expressed in the thickening of the viscous sublayer. In this case for the dimensionless thickness of the viscous sublayer in a flow with polymer additives we can write

\[
\delta^+ = \alpha_1 \overline{A}, \quad \alpha_1 = 11.6.
\]

(9)

We prescribe the profiles of the averaged velocity in the region of the viscous sublayer and the turbulent kernel of the flow as follows:

\[
\bar{u}^+ = y^+, \quad 0 \leq y^+ \leq \alpha_1 \overline{A}; \quad \bar{u}^+ = \chi^{-1} \ln y^+ + c, \quad y^+ \geq \alpha_1 \overline{A}.
\]

(10)

Here

\[
c = \alpha_1 \overline{A} - \chi^{-1} \ln(\alpha_1 \overline{A})
\]

(11)

is a constant of integration, determined from the condition of joining of the velocity profiles (10) on the boundary of the viscous sublayer with \( y^+ = \alpha_1 \overline{A} \); \( \bar{u}^+ = \bar{u}/u_* \) is the dimensionless velocity; \( y^+ = u_* y/\nu \) is a dimensionless coordinate; and \( \chi = 0.4 \).

The coefficient of hydrodynamic resistance of the flow with polymer additives can be computed from the relations

\[
\lambda = 8(u_*/u_p)^2 = 8 \left[ u_*/(u_p^I + u_p^{II}) \right]^2;
\]

\[
u_p^I = \int_0^{\delta/\alpha} u(1 - y/\alpha) d(y/\alpha);
\]

\[
u_p^{II} = \int_{\delta/\alpha}^1 u(1 - y/\alpha) d(y/\alpha).
\]

(12)