SOIL MECHANICS

PREDICTION OF THE STRESS STATE IN A SOIL MASS OF LIMITED THICKNESS AND WIDTH UNDER LOCAL LOADING

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Expressions in the form of series are derived for determination of the stress distribution in a given soil mass of limited thickness and width under a uniformly distributed load and a load varying sinusoidally for the case of the plane problem. The stress state is compared between soil masses of different thicknesses and widths and a soil mass in the form of a half-space. It is demonstrated that they converge with increasing layer thickness and width of the soil mass.

Selection of the computational model of the soil bed is most critical for prediction of bed settlements for structures. Design and in-service experience gained with structures indicates that the computational model of a half-space yields discrepancies in settlement values. At the present time, there are a number of proposals concerning use of the computational model of a mass of limited thickness [1-6].

The stress-strain state of a soil bed under a local loading has definite specifics dictated by the structural strength of the soil. There is a certain structural region within which the stresses exceed the structural strength, and significant deformation of soil occurs. The soils deform negligibly beyond the limits of this stress region. The shape and size of this region depends on the mechanical properties, structural strength, and the area over which the load acts.

Certain authors suggest that this region is closed and assumes the shape of a bulb (Fig. 1), i.e., is limited with respect to both depth and width [1-6].

Analyzing existing computational bed models, it is possible to conclude that a computational model that enables us to account for the boundedness of the region of deformation with respect to both depth and width corresponds most closely to the physics of the phenomenon. For this region, we selected a computational model in the form of a rectangle for the case of the plane problem (see Fig. 1). The dimensions of the region can be determined in first approximation by comparing the structural strength of the soil with the stresses as defined from the computational model of a half-plane. This problem is the separate subject of investigation, and is not addressed in this study. There are specific proposals for determination of the dimensions of the boundary of the deformable region using structural strength and the load-surface concept in the theory of plastic flow [3]. The stress state of a soil of limited thickness and width under a local loading is considered below within the framework of the plane problem of the theory of elasticity. The question concerning the stress distribution in the soil mass for the case of the plane problem (with respect to deformation) is reduced to determination of the stress components \( \sigma_x, \sigma_z, \) and \( \tau_{xz} \) at any point in the mass on the assumption that the stress state varies negligibly over time and corresponds to the solution of the theory of elasticity.

The initial differential equations are written in the following form:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0; \quad \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = 0; \quad \frac{\partial^2 \sigma_{cp}}{\partial x^2} + \frac{\partial^2 \sigma_{avg}}{\partial z^2} = \nabla \sigma_{avg} = 0, \tag{1}
\]

where \( \nabla \) is a Laplace operator, and \( \sigma_{avg} = (\sigma_x + \sigma_z)/2 \) is the average stress.
Fig. 1. Region of soil bed with disturbed structural strength (diagram).

The first two equations of system (1) are equilibrium equations where bulk forces are disregarded, and the last one is a continuity equation expressed in terms of the stresses, which are related to the strain components using physical and geometric equations. It follows from the theory of elasticity [4, 6], and also from system of Eqs. (1) that the stress components can be written in the following form:

\[
\sigma_z = \sigma_{avz} z \frac{\partial \sigma_{avz}}{\partial z}; \quad \sigma_x = \sigma_{avz} + \frac{\partial \sigma_{avz}}{\partial x}; \quad \tau_{xz} = -z \frac{\partial \sigma_{avz}}{\partial x}
\]

Thus, the problem is reduced to solution of the Laplace equation

\[
\nabla \sigma_{avz} = \frac{\partial \sigma_{avz}}{\partial x^2} + \frac{\partial \sigma_{avz}}{\partial z^2} = 0.
\]

For the boundary conditions

\[
\frac{\partial \sigma_{avz}}{\partial x}(0, z) = 0; \quad \frac{\partial \sigma_{avz}}{\partial z}(l, z) = 0;
\]

\[
\sigma_x(x, 0) = \sigma_x(x, 2h) = q \quad \text{for} \quad 0 \leq x \leq +c.
\]

\[
\sigma_z(x, 0) = q \quad \text{and} \quad \sigma_z(x, 2h) = q \quad \text{when} \quad c \leq x \leq +c.
\]

Solving Eq. (3) with allowance for the assigned boundary conditions by the method of separation of variables using Fourier series, we obtain the following expressions for the stress components:

\[
\sigma_{avz} = \frac{\sigma_x + \sigma_z}{2} = q \left[ \frac{c + 2}{l} \pi \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{l} z + \sin \frac{n \pi}{l} (2h - z)}{l} \left( \frac{2n \pi}{l} \right) \frac{\sin \frac{n \pi}{l} (2h - z)}{l} \right] \frac{\sin \frac{n \pi}{l} \cos \frac{n \pi}{l} \cos \frac{n \pi}{l}}{l}
\]

\[
\sigma_x = q \left[ \frac{c + 2}{l} \pi \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{l} z + \sin \frac{n \pi}{l} (2h - z)}{l} \left( \frac{2n \pi}{l} \right) \frac{\sin \frac{n \pi}{l} (2h - z)}{l} \right] \frac{\sin \frac{n \pi}{l} \cos \frac{n \pi}{l} \cos \frac{n \pi}{l}}{l}
\]

\[
\sigma_z = q \left[ \frac{c + 2}{l} \pi \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{l} z + \sin \frac{n \pi}{l} (2h - z)}{l} \left( \frac{2n \pi}{l} \right) \frac{\sin \frac{n \pi}{l} (2h - z)}{l} \right] \frac{\sin \frac{n \pi}{l} \cos \frac{n \pi}{l} \cos \frac{n \pi}{l}}{l}
\]

\[
\tau_{xz} = \frac{2q}{l} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{l} z + \sin \frac{n \pi}{l} (2h - z)}{l} \left( \frac{2n \pi}{l} \right) \frac{\sin \frac{n \pi}{l} (2h - z)}{l} \frac{\sin \frac{n \pi}{l} \sin \frac{n \pi}{l} \cos \frac{n \pi}{l}}{l}; \quad \sigma_y = \mu(\sigma_x + \sigma_z); \quad \sigma_{avz} = \frac{\sigma_x + \sigma_z + \sigma_y}{3} = \frac{2}{3} \mu(1 + \mu)\sigma_{avz},
\]