DESIGN OF SHORTCUT TESTS FOR PRODUCTS WITH INCREASING FAILURE INTENSITY FUNCTION

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For binomial test design when only the total number of tested specimens and the number of failed ones are observed, a model is proposed showing how the lower confidence bound for the probability of reliable functioning in the forced mode can be interpreted in the nominal mode, which allows one to reduce considerably the extent of tests and the testing duration.

The design of tests to control the reliability involves [1] the determination of the sample size and of the duration of testing of an experimental product which are sufficient to make a decision whether the reliability of the products from a batch conforms to a given value. Testing whether the products from a batch conforms to reliability requirements is carried out on the basis of the verification of the inequality [2, 3]

\[ R_\gamma \leq R_R, \]  

where \( R_R \) is the required value of the reliability parameter \( R \), which hereafter means the probability of reliable functioning (PRF) of the product in a given time \( t_0 \) in a given mode \( \varepsilon_0 \); let \( R_\gamma \) be the lower confidence bound (LCB) with the confidence level \( \gamma \) for \( R \) which is calculated from the test results.

One of the most important problems in reliability theory is the problem of shortening the duration of testing (in comparison with the value \( t_0 \)), i.e., the design of shortcut tests at the expense of choosing a more arduous ("forced") mode \( \varepsilon_* \) and constructing a model of interpretation of the results of the tests with regard to the mode \( \varepsilon_0 \). The common approach to the design of shortcut tests consists in the determination of some "quickening coefficient" \( k(\varepsilon_0, \varepsilon_*) \) which allows us to transform the results of the tests in the mode \( \varepsilon_* \) into the results of the tests in the mode \( \varepsilon_0 \) and then use the usual test design methods. But sometimes this approach is not acceptable: for example, in the case where binomial trials are carried out and only the total number \( n \) of tested specimens in time \( t_0 \) and the number of failed ones are observed. Note that the method just mentioned is widely used. In the present paper, for the mentioned test design we construct a model how the LCB \( R_\gamma \) for the PRF of a product in the forced mode can be interpreted in the nominal mode, which allows us to reduce considerably the extent of tests as the testing duration \( t_0 \) and the sample size are reduced.

Let us formulate the problem and initial assumptions. The following two assumptions lie at the heart of the results obtained.

1. The distribution functions of the time before failure of the product of a given batch for a given value of stress coincide up to scale transformation (linear model).
2. The family of distribution functions of the time before failure belongs to the class of distributions with the failure intensity function increasing on the average [4, 5].

Let us analyze these assumptions. We denote by \( T(z) \) the time before failure, and by \( R(z, t) \), the PRF of the product in time \( t \) under the loading \( z \in E_0 \subseteq \mathbb{R} \). Here \( E_0 \) is the set of feasible values of the nonrandom variable \( z \) which determines the loading and therefore the test mode \( \varepsilon_\cdot \). Thus, for the sake of simplicity, here we consider the case where the test mode \( \varepsilon_\cdot \) is determined by a scalar parameter \( z \). The set of feasible values \( E_0 \) of the loading is determined by means of the condition that the processes of exhaustion of the product's resources for different loading levels \( z \in E_0 \) are automodelling ones.

Assumption 1 appears in most methods of shortcut testing [4, 6, 7] and is referred to as the linear model. According to this model [8], there exists a positive monotone increasing function \( r(z) \) such that for all \( z \in E_0 \) the distribution of the random variable \( r(z)T(z) \) remains the same and does not depend on \( z \). This assumption is equivalent to the assumption that variations of the test mode affect only the scale parameter of the family of distributions \( F(z, t) \), which can be formally written as

where \( R(x, t) = 1 - \Phi(x, t) \).

The function \( r(x) \) is interpreted as the rate of exhaustion of the product’s resources under the loading \( x \). The form of the function \( r(x) \) can be deduced from the functional relation between the reliability characteristics and the loading parameters. For example, if such a relation exists for the mean time before failure (MTBF) \( T_m(x) = E T(x) \) in the form \( T_m(x) = A \Phi(x) \), where \( A \) does not depend on \( x \), then

\[
R(x, t) = 1 - \Phi(x, t).
\]

The second assumption means that for any two values \( t_1, t_2, t_1 \leq t_2 \), the inequality [5]

\[
R(x_0, t_0 | n, d) \geq R(x_1, t_1),
\]

holds. The presence of relation (2) and inequality (3) allows us, by forcing the loading \( x \), i.e., by passing from \( x_1 \) to \( x_2 > x_1 \), \( x_1, x_2 \in E_0 \), to shorten the test duration and, generally speaking, the sample size. Let us show how the test design is carried out in the normal mode, without forcing [1, 2]. Under binomial trials, one observes a sample of products of size \( n \) in time \( t_0 \), where \( t_0 \) is the given time of functioning of the product, and also the number \( d \) of failed products. By these data, one can determine, by the Clopper-Pearson equations [1, 3], the LCB with level \( \gamma \) in the form

\[
R_\gamma(x_0, t_0 | n, d) = f_2(n, d, \gamma),
\]

where \( x_0 \) is the loading during exploitation, and \( f_2(n, d, \gamma) \) is the function tabulated in [3].

There exists a rather good approximation of the function \( f_2 \) [2]:

\[
R_\gamma(x_0, t_0 | n, d) \approx 1 - \frac{x_0^2(2d + 1)}{2n - d + \frac{3}{2}x_0^2(2d + 1)},
\]

where \( x_0^2(\nu) \) is the quantile of the chi-squared distribution with \( \nu \) degrees of freedom. In the case of unfailing tests, i.e., where \( d = 0 \), the expression for \( R_\gamma \) has the following simple form:

\[
R_\gamma(x_0, t_0 | n, 0) = (1 - \gamma)^{1/n}.
\]

Knowing the expression for \( R_\gamma \) and setting an allowable number of failures, one can derive from inequality (1) the required sample size \( n \). In particular, (6) implies that the minimum size of unfailing test \( (d = 0) \) is expressed as

\[
n_0 \geq \frac{\log(1 - \gamma)}{\log \Phi_R}.
\]

Let us formulate the basic results.

**Theorem 1.** Let \( x_1, x_2 \in E_0 \subset \mathbf{R} \), let the linear model (2) hold, and let the reliability functions \( R(x_1, t), R(x_2, t) \) belong to the class of distributions with the failure intensity function increasing on the average. Then for \( t_1 \) and \( t_2 \) such that \( r(x_1)t_1 \leq r(x_2)t_2 \) the inequality

\[
[R(x_2, t_2)]^\kappa \leq R(x_1, t_1), \quad \kappa = \kappa_r \kappa_t = \frac{r(x_1)t_1}{r(x_2)t_2},
\]

is valid.

**Proof.** Based on relation (2), we have

\[
R(x_1, t_1) = R\left( x_2, \frac{r(x_1)t_1}{r(x_2)t_1} \right).
\]

The presence of the condition \( r(x_1)t_1 \leq r(x_2)t_2 \) allows us to rewrite inequality (3) as

\[
R\left( x_2, \frac{r(x_1)t_1}{r(x_2)t_1} \right) \geq [R(x_2, t_2)]^{r(x_1)t_1/r(x_2)t_2}.
\]

Relations (9) and (10) yield the assertion of the theorem.