ON THE DETERMINATION OF THE PARAMETERS THAT DESCRIBE A QUASISTATIC ELECTROMAGNETIC FIELD IN AN ELECTRICALLY CONDUCTING SHELL

O. R. Gachkevich and B. I. Chornii

We state the initial equations and the boundary conditions for determining the parameters that describe a quasistatic electromagnetic field perturbed by the action of external electric currents in a thin nonferromagnetic shell.

Consider a thin electrically conducting shell of constant thickness $2h$ in a region $D$ in which there is no charge or current. The shell is situated in a dielectric medium $D_0$ (which we assume to be approximately a vacuum) and subject to the action of a quasistatic electromagnetic field. The field is generated by a system of quasistatic currents flowing in $D_0$ (an inductor of time-varying intensity), given by the expression

$$j^{(0)}(r,t) = j(t)f(r)\cos(\omega t + \phi_0), \quad \text{div} \, f(r) = 0,$$

(1)

where $r$ is the radius-vector of the point, $\omega$ is the cyclic frequency, $t$ is time, $\phi_0$ is the initial phase, and $j_A(r,t) = j(t)f(r)$ is the amplitude, which varies only slightly over the period of an electromagnetic wave, $f_r = 2\pi i/\omega$, so that the following condition holds:

$$|dj(t)/dt| \ll \omega |j(t)|.$$

(2)

We pose the problem of determining the parameters that describe the electromagnetic field under such an action in the shell-external medium system. We recall that finding these parameters is the first step in computing the thermoelastic state of electrically conducting shells perturbed by an electromagnetic field [1].

We shall start from Maxwell's equations for the external medium (a vacuum) and a solid isotropic nonferromagnetic nonpolarized motionless body with constant (averaged over a temperature interval) material characteristics and neglecting the displacement currents [5]. We write the following system of independent equations with complex current density $j_r = j(t)\text{Re} \{j(r)e^{i\omega t}\}$, electric field intensities $E_r = j(t)\text{Re} \{E(r)e^{i\omega t}\}$, $E^{(0)} = j(t)\text{Re} \{E^{(0)}e^{i\omega t}\}$, and magnetic field intensities $H_r = j(t)\text{Re} \{H(r)e^{i\omega t}\}$, $H^{(0)} = j(t)\text{Re} \{H^{(0)}e^{i\omega t}\}$ in the region $D$ of the body and $D_0$ of the surrounding medium, taking account of relations (1) and (2) (see [1]):

$$\text{curl} \, H = \sigma E, \quad \text{curl} \, E = -i\mu_0 \omega H, \quad \text{curl} \, H^{(0)} = i\varepsilon_0 \omega E^{(0)} + j^{(0)}, \quad \text{curl} \, E^{(0)} = -i\mu_0 \omega H^{(0)}.$$

(3)

Here $j^{(0)} = f(r)e^{i\phi_0}$ is the complex amplitude of the given current density in the surrounding medium; $E_r, H_r, E^{(0)}$ and $H^{(0)}$ are the complex amplitudes of the intensities of the electric and magnetic fields in the regions $D$ and $D_0$ respectively; $\varepsilon = \varepsilon_0 \varepsilon_s$, $\mu = \mu_0 \mu_s$, and $\mu_0$ are the dielectric permittivity and magnetic permeability of the body and the magnetic permeability of the vacuum; $\sigma$ is the electrical conductivity of the body.

Equations (3) must be supplemented by the independent coupling conditions of the characteristics of the electromagnetic field at the interface between the body and the surrounding medium:

$$(E)_T = (E^{(0)})_T, \quad (H)_T = (H^{(0)})_T.$$

(4)
and also the conditions for radiation at infinity in the region \( D_0 \) [5]. The subscripts \( \tau \) and \( n \) denote the tangential and normal components of the respective vectors on the surface \( S \).

Equations (3) can be reduced by a known method [1, 5] to a system of equivalent equations in the functions \( E, E^{(0)} \) or \( H, H^{(0)} \). Such a system in \( E \) and \( E^{(0)} \) has the form

\[
(\Delta + \kappa^2)E = 0, \quad \text{div} \, E = 0, \quad (\Delta + \kappa_0^2)E^{(0)} = i\mu_0 \omega f^{(0)}, \quad \text{div} \, E^{(0)} = 0,
\]

where \( \Delta \) is the Laplacian, \( \kappa^2 = -i\mu\sigma\omega \), \( \kappa_0^2 = \omega^2 c_0^{-2} \), and \( c_0 = (\varepsilon_0 \mu_0)^{1/2} \) is the speed of propagation of electromagnetic waves in a vacuum [5]. The coupling conditions (4) are written as follows:

\[
(E)_{\tau} = (E^{(0)})_{\tau}, \quad \frac{1}{\mu}(\text{curl} \, E)_{\tau} = \frac{1}{\mu_0}(\text{curl} \, E^{(0)})_{\tau},
\]

and the condition of radiation at infinity [5], taking account of the data in [6], is

\[
\lim_{r \to \infty} r \left( \frac{\partial}{\partial r} + \frac{i \omega}{c_0} \right) E^{(0)}(r) = 0.
\]

The functions \( H, H^{(0)} \), and \( E, E^{(0)} \) here are connected by the relations

\[
H = -\frac{1}{i\mu\omega} \text{curl} \, E, \quad H^{(0)} = -\frac{1}{i\mu_0 \omega} \text{curl} \, E^{(0)}.
\]

The problem (5)-(7) is equivalent to the problem described by the equations [1]

\[
(\Delta + \kappa^2)E = 0, \quad (\Delta + \kappa_0^2)E^{(0)} = i\mu_0 \omega f^{(0)}
\]

with the following coupling conditions on the surface \( S \):

\[
(E)_{\tau} = (E^{(0)})_{\tau}, \quad \frac{1}{\mu}(\text{curl} \, E)_{\tau} = \frac{1}{\mu_0}(\text{curl} \, E^{(0)})_{\tau}, \quad \sigma(E)_{n} = i\varepsilon_0 \omega (E^{(0)})_{n}, \quad \text{div} \, E = \text{div} \, E^{(0)},
\]

and the radiation condition (7) at infinity (in the region \( D_0 \)).

Similarly we obtain the key system of equations in the functions \( H \) and \( H^{(0)} \):

\[
(\Delta + \kappa^2)H = 0, \quad (\Delta + \kappa_0^2)H^{(0)} = -\text{curl} \, f^{(0)}
\]

with the following coupling condition on \( S \):

\[
(H)_{\tau} = (H^{(0)})_{\tau}, \quad \frac{1}{\sigma}(\text{curl} \, H)_{\tau} = \frac{1}{i\varepsilon_0 \omega}(\text{curl} \, H^{(0)})_{\tau}, \quad \mu(H)_{n} = \mu_0 (H^{(0)})_{n}, \quad \text{div} \, H = \text{div} \, H^{(0)},
\]

with the following radiation condition at infinity:

\[
\lim_{r \to \infty} r \left( \frac{\partial}{\partial r} + \frac{i \omega}{c_0} \right) H^{(0)}(r) = 0.
\]

When problems are stated in terms of the functions \( H \) and \( H^{(0)} \), the complex intensities \( E \) and \( E^{(0)} \) are expressed as follows:

\[
E = \frac{1}{\sigma} \text{curl} \, H, \quad E^{(0)} = \frac{1}{i\varepsilon_0 \omega} \text{curl} \, (H^{(0)} - f^{(0)}).
\]

We remark that the relations given above can also be taken as the initial relations for problems of determining the electromagnetic field characteristics in a ferromagnetic electrically conducting body in an engineering approximation. To do this one must replace the parameter \( \mu_* \) by the effective magnetic permeability \( \mu_\varepsilon \) [1].

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