MATHEMATICAL MODELING OF THE KINETICS OF WEAR-FREE FRICTIONAL PROCESSES

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We propose a version of a mathematical model for the quantitative description of a frictional process involving substances that are able to restore the working surfaces. In doing so we take account of the processes of ordinary friction accompanied by fracture and an increase in the contacting surfaces, as well as wear-free friction processes. We establish that the transition from the one kinetic process to the other passes through a bifurcation point separating the stable and unstable solutions of the proposed system of equations.

The present paper relates to a rather large class of phenomena characterized by the presence of intensive exchange of energy and/or mass with the surrounding medium at a metal surface/medium interface. In particular the subject may be frictional processes, or the treatment of metal surfaces by ultrasound or high-power coherent radiation. All these processes of action on the surface may be accompanied by fracture of the surface layer with evolution specific to that layer. The phenomena and the kinetics of the evolutionary fracture of tribosystems have been studied in great detail in the literature [3, 6, 7].

In contrast to the traditional solution of such problems, based on the phenomenon of selective transport of copper [4] or structural adaptation of the materials to the frictional conditions [6], our approach is based on new pictures of physico-chemical processes that provide wear-resistance to the friction surfaces, which has been verified experimentally [1]. Based on these pictures, workers at the Institute of General and Inorganic Chemistry of the Ukrainian National Academy of Sciences have synthesized a class of substances which have been classified as extremely effective additives by independent experts, greatly increasing the capacity of the lubricating materials [2]. The significant decrease in wear achieved as the result of tribochemical reactions involving such substances, was described in [5].

In the present paper we attempt a mathematical description of the kinetics of anti-wear friction [1] taking account of the conditions of fracture and restoration of the surface. In doing so we shall assume that the change in the contact surface at the metal surface/medium interface due to friction takes place as follows. At the initial stage of the process fracture is determined by the number of broken chemical bonds and is proportional to the area of contact. In the presence of pressure (load) it is also necessary to take into account that the rate of change of the area of contact increases in proportion to the pressure applied. These two factors lead to an increase in the area of contact during the frictional process. When matter returns to the surface, a partial restoration of the fractured surface occurs in which the contact surface decreases—a thin film is observed to form on the surface.

On the basis of experimental data we assume that the formation of a stable surface occurs as the result of the natural removal of matter from the surface (fracture to the surface layer) and the return of matter to the surface (restoration of the surface). The properties of the organizing substances are chosen so that their rate of "deposit" on the friction surface exceeds the rate of removal. The behavior of the system depends strongly on the structure of the organizing substances. The formation of a thin surface layer in the shape of the original relief has been verified experimentally, from which it follows that the rate of deposit of the organizing substance on itself and on the "pure" surface (free of organizing substance) differ considerably. The role of the organizing substances is also vital in this process because by varying their concentration one can restore the balance between the two processes that form the surface layer (removal and deposit of matter), i.e., one can influence the course of the process being studied.

We denote the amount of organizing substance in the surface layer by \( x(t) \). Then its concentration is \( x(t)/\delta S(t) \), where \( \delta \) is the mean thickness of the film. The concentration of vacancies in the film is defined
as \( c_0 - x(t)/\delta S(t) \), where \( c_0 \) is the limiting concentration of organizing substance in a film consisting only of the organizing substance.

The time variation of the area of contact \( S(t) \) can be described by the following kinetic equation:

\[
\frac{dS(t)}{dt} = \sigma S(t) - \Delta_1 S(t)c(t)[c_0 - x(t)/\delta S(t)] - \Delta_2 S(t)x(t) + \alpha/S(t).
\]  

(1)

Here the coefficient \( \sigma \) characterizes the susceptibility of the surface to fracture, and the parameters \( \Delta_1 \) and \( \Delta_2 \) are the probabilities of depositing the organizing substance on the “pure” surface and on itself. The last term in Eq. (1) describes the increase in the area of the contact surface under pressure (at pressures below the yield limit).

In describing Eq. (1) we assume that the probability of depositing the organizing substances on a surface is proportional to the concentration \( c(t) \) of the organizing substance in the solution and the surface area. We describe the variation of the quantity of organizing substance in the surface layer by the following kinetic equation:

\[
\frac{dx(t)}{dt} = S(t)c(t)[c_0 - x(t)/\delta S(t)] + \chi c(t)x(t)/\delta - \gamma x(t).
\]  

(2)

The first term on the right-hand side of Eq. (2) describes the increase in the substance under partial restoration of the fractured surface, as in Eq. (1). For convenience the value of the rate of deposit of organizing substance on the “pure” surface in Eq. (2) is taken equal to 1; \( \chi \) is the rate of deposition of organizing substance on itself. The last term describes the removal of matter from the surface, including the action of a load \( F \). Thus the quantity \( \gamma \) should increase as the pressure increases.

In the system under consideration both chemical and physical processes take place. The process of forming a surface layer can be regarded as adiabatic in this case, since the chemical processes on the surface, as well as the migration processes in the medium are rapid, lasting a much shorter time than the experimental observables [1]. This leads to a situation in which an ordinary chemical equilibrium establishes itself near the metal/medium interface under which the concentrations of organizing substance in the medium and in the surface layer are proportional. Their ratio is determined by the values of the coefficients of dissolution of matter from the surface and deposition on the surface. Thus in considering relatively slow triboprocesses one may assume that \( c(t) = \zeta x(t)/\delta S(t) \), where \( \zeta \) is a constant.

Let us consider the solution of Eqs. (1) and (2). The coordinates of the stationary point \( x_0, S_0 \) are defined as follows:

\[
S_0 = \sqrt{\sigma}/[\Delta_1 c_0 - \gamma_0 - \gamma_0/\zeta - \sigma]^{1/2}, \quad x_0 = (c_0 - \gamma_0/\zeta)\delta S_0.
\]  

(3)

Numerical computations showed that the evolution of the unstable solutions that describe the behavior of the system near a nontrivial stationary point into stable solutions passes through a point of bifurcation. The critical value \( \gamma_c \) that characterizes the removal of matter from the surface is defined by the expression

\[
\gamma_c = \frac{1}{1 + \Delta_2 c_0 - 2\sigma} (\zeta c_0/\delta + \Delta \zeta c_0^2 - 1).
\]  

(4)

The behavior of the solutions of Eqs. (1) and (2) is shown in the figure. At the critical point the solutions \( S(t) \) and \( x(t) \) (curves (1)) oscillate about the corresponding steady values with frequency

\[
\omega_c = \zeta/\delta(c_0 - \gamma_0/\zeta) [2\Delta_2 c_0^2/\zeta - \Delta \zeta c_0^2 - 1]^{1/2}.
\]  

(5)