DYNAMIC MODELING OF ELECTRON-RAY DEVICES
BY THE METHOD OF LARGE PARTICLES

M. V. Glumova and A. A. Shadrin

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We propose a dynamic model of the electron projector of an electron-ray tube, developed using the method of large particles and using statistical methods for modeling the cathode.

In recent years the method of large particles has received widespread application in numerical modeling of motions characterized by large deformations, displacements of the medium, nonstationarity, and nonlinearity of the processes that occur in the mechanics of solid media.

This method makes it possible to obtain the dynamics of the evolution of the phenomenon, the characteristic properties of the currents that arise in the medium when the interval of variation of the medium itself is large, the shape of the body, and other parameters. At the same time applications of electron optics in microelectronics, diagnostics of materials, and the treatment of surfaces in connection with the development of technology using sharply focused electron beams of various powers, all require a dynamic description of the processes in electron projectors and electron-optic systems. The method of large particles happens to be effective for solving this class of problems; it was the method applied to create the dynamic model. The papers [1–3] give an application of the method of large particles for intensive electron beams in modeling klystrons, magnetrons, transport systems, and other electrophysical devices.

The basis of the mathematical model in our case was the solution of a self-consistent problem that presumes a solution of the system of equations of motion of particles subject to the three-dimensional charge fields created by the particles themselves and external electrostatic fields determined by solving the Poisson and Laplace equations respectively. The electron flux that is modeled was replaced by a group of macroparticles with an automatically varying enlargement factor coinciding at a given instant of time with the cells of an imposed $N \times N$ Euler grid. The computation process consists of repeated steps on time, at each of which the fields being accounted for were computed with a stationary grid; in solving the equations of motion a redistribution of particles in space was carried out, i.e., new coordinates and velocities of the particles were determined using their values at the preceding step, and the anode current, variation of the flux, leakage currents, and the geometric and current characteristics of the beam were found. In determining the parameters of the model that effect the discretization of space and time—the space-time step, it was assumed that the distance traversed by a particle in one time step did not exceed the step in the space grid. The time step remained constant during the count of the entire model; it was chosen from the condition of precision in the solution of the equations of motion and was written in accordance with the geometric dimensions of the region being studied and the average velocity of the particles in the volume. The space step varied over the whole length of the electron-optical system depending on the number of particles in the various regions of the volume being studied.

The original system of equations is a set of equations for determining the electric fields and the Newtonian equations of motion written in cylindrical coordinates:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2} + \frac{\partial^2 U}{\partial z^2} = F(r, \varphi, z),$$

$$\frac{d(mr)}{dt} - mr(\varphi)^2 = qE_r, \quad \frac{1}{r} \frac{d(mr^2 \varphi)}{dt} = qE_\varphi, \quad \frac{d(mz)}{dt} = qE_z, \quad (1)$$

where $F(r, \varphi, z) = \frac{\rho(r, \varphi, z)}{\varepsilon_0}$, $\rho$ is the charge density at the given point of the beam, and $E_r$, $E_\varphi$, and $E_z$ are the components of the electric fields.

The equations of motion were written relative to the center of the electron clouds of particles whose volume is respectively equal to the dimension of a cell of the space grid. After passing to a rectangular grid and the change of variables

\[ r = r_k e^{k_y Y}, \quad \varphi = k_x X, \quad k_x = \frac{2\pi}{M_x}, \quad k_y = \frac{\ln r_a/r_k}{M}, \quad k_z = \frac{L}{N}, \]  

(2)

where \( r_a \) is the radius of the bulb of the electron projector, \( r_k \) is the radius of the first tier of the grid axis of the electron-optical system, \( M \) is the number of partitions with respect to \( Y \), \( M_x \) is the number of partitions with respect to \( x \), \( L \) is the length along the \( Z \)-axis, \( N \) is the number of partitions with respect to the \( Z \)-axis, and also after application of the one-dimensional fast Fourier transform on the coordinate in combination with the finite-difference scheme for the equation for computing the fields, the Adams extrapolation formulas for the velocities, and power series in the equations of motion, the working system of equations assumed the following form:

\[
\begin{align*}
U_i^{t+1} = & \left( 2 + \beta_i \left( \frac{2\pi j}{N} \right)^2 \right) U_i^t + U_i^{t-1} = -\frac{k_y}{2\varepsilon_0 k_z Q_i^{t+2}}, \\
z_{k+1} = & z_k + V_{z,k}\Delta t + \frac{\Delta t^2}{4} \left( \frac{19}{6} G_n - \frac{5}{3} G_{k-1} + \frac{1}{2} G_{k-2} \right), \\
y_{k+1} = & y_k + V_{y,k}\Delta t + \frac{\Delta t^2}{4} \left( \frac{19}{6} G_n - \frac{5}{3} G_{k-1} + \frac{1}{2} G_{k-2} \right), \\
V_{z,k+1} = & V_{z,k} + \frac{\Delta t}{3} \left( \frac{23}{4} G_k - 4G_{k-1} + \frac{5}{4} G_{k-2} \right), \\
V_{y,k+1} = & V_{y,k} + \frac{\Delta t}{3} \left( \frac{23}{4} G_k - 4G_{k-1} + \frac{5}{4} G_{k-2} \right), \\
\beta_i = & k_y r_k \exp(k_y Y),
\end{align*}
\]

(3)

here \( U_i^{t+1} \) are the cosine and sine harmonics of the potential, \( Q_i^{t+2} \) are the cosine and sine harmonics of the charge, \( z_{k+1}, y_{k+1}, V_{z,k+1}, \) and \( V_{y,k+1} \) are the coordinates and velocities at time step number \( k + 1 \); \( G_k, G_{k-1}, \) and \( G_{k-2} \) are the accelerations.

In solving the system for the equation that determines the electrostatic fields we applied the method of the capacity matrix in conjunction with the method of solving the Poisson equation by using the fast Fourier transform with Gaussian elimination in (1).

The work of the model began immediately after the introduction of the macroparticles from the cathode or the modeling of the emission in the cathode block of the electron-ray tube. As noted in [3], the automation in the projection of the cathode-heating node is practically absent. This is because many physical properties and phenomena occurring in oxide cathodes (local heating of the oxide coatings as a result of the passage of significant currents, oscillation of the emission capability of the cathode and the rate of evaporation of the substance of the oxide coatings, the material of the cathode base, the doping processes at the cathode) have not received an unambiguous interpretation. It is therefore reasonable for a numerical description of the working of this node to apply the methods of statistical modeling using random number generators—Monte Carlo methods. The particles emitted from the cathode had a Poisson distribution

\[ \mathcal{R}(n \leq m) = \sum_{n}^{m} P_n; \quad P_n = p_n e^{-\bar{n}}; \quad p_n = \frac{n}{n!}, \quad (n = 0, 1, 2, \ldots), \]

(4)

where \( P_n \) is the probability that the random variable assumes the value \( n \), \( \mathcal{R}(n \leq m) \) is the probability that the random variable assumes a value \( n \leq m \), \( \bar{n} \) is the mean value of the integer-valued random variable: \( \bar{n} = I dt/Q = jS dt/Q \) (\( I \) is the current density, \( Q \) is the charge of an enlarged particle), determined in