A HYBRID METHOD OF STUDYING THE STRESS INTENSITY FACTORS AT THE TIPS OF CRACKS IN ANISOTROPIC PLATES

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We propose an efficient new method of numerical analysis of the residual strength of finite plates weakened by cracks. The basis of the method is an alternating Schwartz procedure that makes it possible to achieve a successful combination of the algorithmicity and indifference to boundary conditions of the finite-element method and the application of the method of integral equations for studying singular stress fields. The numerical implementation of the scheme and the algorithm for determining the stress intensity factors at the tips of cracks is given as a software package written in FORTRAN. The efficiency of application of the computational methodology is illustrated by examples.

3 Figures. 4 Tables. Bibliography: 6 titles.

Determination of the stress intensity factors at the tips of slits of arbitrary shape in finite anisotropic plates encounters certain difficulties even when such efficient numerical methods as the integral equation method and the finite-element method are applied. In the case of the finite-element method, despite its universality and algorithmic nature, simplicity of solution for regions of complicated shape, and indifference to boundary conditions, the difficulties are connected with the precise analysis of the singularity of the stress fields. The successful application of the method of integral equations in studying singular stress fields is complicated by the finite shapes of the body and the irregularities of the boundaries. The use of special finite elements or the solution of the additional integral equations on the boundary of the body does not completely free the methods from these defects, yet the cumbersomeness of the computations increases significantly.

The essence of the proposed approach, which preserves the advantages of the finite-element method and the method of integral equations consists of combining them using the alternating Schwartz method of [1]. Its basic idea and computations are simple, but to date it has found few applications to the solution of practical problems.

Consider an anisotropic plate occupying the region $D$ with boundary $C$ in the $xy$-plane. The plate is weakened by a curvilinear slit $L$. A system of external forces is applied to the boundary $C$:

$$F(t) = \sigma(t) + i\tau(t), \quad t \in C,$$

where $\sigma$ and $\tau$ are the normal and tangential components of the vector $F$. The edges of the slit $L$ are free of load.

We now break the problem into two parts (Fig. 1) and construct two sequences of corresponding solutions. In the first of these we exclude the slit and, using the finite-element method, we obtain a sequence of solutions (A) for the undamaged plate (the region $D_0$) in the form of the forces

$$\{p_j(t)\} = \{-[\sigma(t) + i\tau(t)]\}$$

on the edges of an imaginary slit $L'$ corresponding to the sequence of loads $F_j(t)$, $(j = 0, 1, 2, \ldots)$. The second sequence of solutions (B) corresponds to an infinite plate (region $D_\infty$) with a real crack $L$ whose edges are loaded by forces $p_j(t)$. These solutions can be obtained in the form of potentials [2] using the method of integral equations

$$\Phi_j(z) = \frac{1}{2\pi i} \int_L \frac{\omega(\tau) d\tau}{\tau - z},$$
where the unknown functions $\omega_\nu(t) \ (\nu = 1, 2)$ are determined from the following system of integral equations

$$
\int L \frac{\omega_1(\tau) \, d\tau_1}{\tau_1 - t_1} + \int L \{K_1(t, \tau) \omega_1(\tau) + K_2(t, \tau) \overline{\omega_1(\tau)}\} \, ds = f(t); \tag{4}
$$

$$
\int L \omega_1(\tau) \, d\tau_1 = 0; \quad \omega_2(t) = -a(t) \omega_1(t) - b(t) \overline{\omega_1(t)}; \quad f(t) = g(t, p_j(t)).
$$

Here we are using the notation of [3].

The stress intensity factors $K_{1,2}^{(j)}$ at the tips of the crack $L$, as well as the values of the forces $[F_j(t)]$ at the points of the imaginary contour $C'$ drawn to conform to the shape of the boundary $C$, can be obtained from the solutions $\Phi_j(z_\nu)$ of the sequence (B).

Naturally the first solution from the sequence (A) in the form of the forces applied to the slit $L$ will not satisfy the conditions of the original problem in the region $D_0$, just as the solution from the sequence (B) will have a discrepancy with the external load of the plate of the region $D_0$. However, if we connect the sequences (A) and (B) by a recursive process in accordance with the alternating Schwartz scheme, the discrepancies on the contours $C$ and $L$ can be minimized, and hence a solution with acceptable precision can be obtained for the problem that was posed. Summing the terms of the sequence of stress intensity factors $K_{1,2}^{(j)}$, we obtain the solution in the form

$$
K_{1,2} = \sum_{j=0}^{\infty} K_{1,2}^{(j)}. \tag{5}
$$

The procedure just described can easily be generalized to a multiconnected region with a system of slits.

If the boundary $C$ of the region $D_0$ contains a rectilinear rim $x = 0$ (Fig. 1) or an arc of an elliptic curve $\Lambda = \{(x/a)^2 + (y/b)^2 = 1\}$, it is reasonable to choose $D_\infty$ as the half-plane $D_\infty = \{x > 0\}$ or the plane with an elliptic slit $\Lambda$ and to use the potentials

$$
\Phi_\nu(z_\nu) = \frac{1}{2\pi i} \int L \left\{ \frac{\omega_\nu(\tau) \, d\tau_\nu}{\tau_\nu - z_\nu} - \frac{l_\nu s_\nu \omega_1(\tau) \, d\tau_1}{\tau_1 - s_\nu z_\nu} - \frac{n_\nu m_\nu \omega_2(\tau) \, d\tau_2}{\tau_2 - n_\nu z_\nu} \right\};
$$

$$
\Phi_{\nu'}(z_{\nu'}) = \frac{1}{2\pi i \omega_{\nu'}(z_{\nu'})} \int L \left\{ \frac{\omega_{\nu'}(\tau) \, d\tau_{\nu'}}{\tau_{\nu'} - z_{\nu'}} - \frac{l_{\nu'} s_{\nu'} \omega_{1'}(\tau) \, d\tau_1}{s_{\nu'}(1 - s_\nu \eta_1)} - \frac{n_{\nu'} m_{\nu'} \omega_{2'}(\tau) \, d\tau_2}{s_{\nu'}(1 - s_\nu \eta_2)} \right\}, \tag{6}
$$

which are constructed in [4] and automatically satisfy the boundary conditions $F_j(t) = 0$ on the rim $x = 0$ or on $\Lambda$. The unknown functions are still determined from the system of integral equations (4).