ELASTIC STRESS CONCENTRATION NEAR BOUNDARY DEFECTS IN THE CASE OF A LONGITUDINAL SHEAR DEFORMATION

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We study the concentration of stresses due to two boundary defects located in an elastic half-space with stress-free boundary loaded at infinity with a constant shear load. The problem is reduced to solving singular integral equations for the cases in which the half-space contains defects of different types: two cracks, two inclusions, and a crack and an inclusion, whose solutions are sought by the method of mechanical quadratures. The interaction of the defects as they approach each other and the influence of their relative sizes are studied numerically.

For the case of antiplanar deformation we use discontinuous solutions of the equations of elasticity theory to reduce problems of elastic stress concentrations near boundary defects located in an elastic half-space in a state of longitudinal shear deformation to systems of singular integral equations whose solutions are constructed numerically by the method of mechanical quadratures. The approximate solution obtained makes it possible to study the influence of the distance between the defects and their sizes on the elastic stress concentration near them.

1. Construction of a discontinuous solution for the equations of elasticity theory in the case of longitudinal shear deformation. We understand a discontinuous solution of the equations of elasticity theory to be a solution for which the displacements and stresses have prescribed jumps at lines (or surfaces) at which defects are located and which decreases at infinity. In the case of a longitudinal shear deformation along the OZ-axis only the Z-component of the displacement vector is nonzero, and in the XY-plane, y > 0, it satisfies Laplace's equation

\[ \Delta w(x, y) = 0. \]  

Suppose that the boundary of the half-space is stress-free:

\[ \tau_{yz} \big|_{y=0} = G \frac{\partial w}{\partial y} \big|_{y=0} = 0 \]  

and that it contains a defect \((-\infty < z < \infty, 0 \leq y \leq a, x = l)\) at which the displacements and stresses have jumps

\[ (w(l, y)) = w(l + 0, y) - w(l - 0, y) = \chi_1(y); \quad (\tau_{xz}(l, y)) = G(w_x'(l, y)) = G\chi_2(y). \]  

According to [8] a discontinuous solution of Eq. (1.1) satisfying (1.2) and (1.3) can be constructed using the Fourier transform and has the form

\[ \omega(x, y) = \int_0^a \chi_2(\eta) [V(x - l, y - \eta) + V(x - l, y + \eta)] d\eta \]

\[ + \int_0^a \chi_1(\eta) \frac{\partial}{\partial x} [V(x - l, y - \eta) + V(x - l, y + \eta)] d\eta, \]

where

\[ V(x, y) = \frac{1}{4\pi} \ln(x^2 + y^2) + \text{const.} \]

Extending $\chi_1(y)$ and $\chi_2(y)$ as even functions to $[-a,0]$, we obtain an even extension of the function $w(x,y)$ to $-\infty < y < 0$. If $\chi_1(\pm a) = 0$, it will be more convenient below to represent the discontinuous solution as

$$w(x,y) = \frac{1}{4\pi} \int_{-a}^{a} \chi_2(\eta) \ln[(x-l)^2 + (y-\eta)^2] d\eta - \frac{1}{2\pi} \int_{-a}^{a} \chi_1'(\eta) \arctan \frac{y-\eta}{x-l} d\eta. \quad (1.4)$$

In the derivation of (1.4) the second integral resulted from integration by parts.

2. Statement of problems of elastic stress concentrations near boundary defects. Consider an elastic half-space $-\infty < x, z < \infty, y \geq 0$, in a state of longitudinal shear deformation and containing two boundary defects in the form of poles $x = \pm l, \pm a \leq y \leq a \pm$. The boundary of the half-space is stress-free, and a constant shear load $\tau_{xz} = P$ is applied at infinity. It is required to construct the stress and displacement fields and determine the interaction of the defects.

Let the defects be two strip cracks whose edges are stress-free. We represent the displacements as a sum of two terms:

$$w(x,y) = w_0(x,y) + w_*(x,y). \quad (2.1)$$

The first solution of the problem in the absence of defects has the form

$$w_0(x,y) = -\frac{P}{G} x + \text{const.}$$

The second term $w_*(x,y)$ is the solution of the problem of cracks to whose edges a load is applied:

$$\tau_{xz}(x = \pm l \pm 0, y) = -P, \quad \tau_{xz}(x = \pm l \pm 0, y) = -P. \quad (2.2)$$

In crossing a crack the stress $\tau_{xz}$ is continuous; only the displacement undergoes a discontinuity. We denote by $\chi_1(\pm y)$ the jumps in displacement at $x = \pm l$, and we represent $w_*(x,y)$ as the sum of two discontinuous solutions constructed according to formula (1.4) with jumps at the lines $x = \pm l$:

$$w_*(x,y) = -\frac{1}{2\pi} \int_{-a}^{a} \chi_1'(\eta) \arctan \frac{y-\eta}{x-l} d\eta - \frac{1}{2\pi} \int_{-a}^{a} \chi_1(-\eta) \arctan \frac{y-\eta}{x+l} d\eta. \quad (2.5)$$

By implementing the boundary condition (2.2) we obtain a system of integral equations in $\chi_1(y)$:

$$\frac{1}{\pi} \int_{-1}^{1} \varphi_{i-}(t) \frac{dt}{\tau - t} + \frac{1}{\pi} \int_{-1}^{1} \varphi_{i-}(t) R_0(\tau - \pi t) dt = f_i(t), \quad |t| \leq 1; \quad (2.3)$$

$$\frac{x}{\pi} \int_{-1}^{1} \varphi_{i+}(t) R_0(x\tau - t) dt + \frac{1}{\pi} \int_{-1}^{1} \varphi_{i+}(t) \frac{dt}{\tau - t} = f_i(t).$$

In the case of two rigid strip inclusions the solution can also be found in the form (2.1), where $w_*(x,y)$ is a solution of the problem of inclusions at which the conditions $w_*(-l \pm 0, y) = \delta, w_*(l \pm 0, y) = \delta$ hold (here $\delta$ is a certain constant whose presence is due to the fact that the shift of the elastic half-space is determined only up to an additive constant). These conditions are obviously equivalent to the following:

$$\frac{\partial}{\partial y} w_*(-l \pm 0, y) = 0, \quad \frac{\partial}{\partial y} w_*(l \pm 0, y) = 0. \quad (2.4)$$

In crossing an inclusion the displacements are continuous: $\chi_1+(y) = \chi_1-(y) = 0$. The jumps in the stresses $\tau_{xz}$ at $x = \pm l$ will be denoted $\chi_2(\pm y)$. We now represent $w_*(x,y)$ as two discontinuous solutions of (1.4):

$$w_*(x,y) = \frac{1}{4\pi} \int_{-a}^{a} \chi_2(\eta) \ln[(x-l)^2 + (y-\eta)^2] d\eta + \frac{1}{4\pi} \int_{-a}^{a} \chi_2(-\eta) \ln[(x+l)^2 + (y-\eta)^2] d\eta.$$

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