A THEORY OF EXCHANGE RATE MODELING

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The article examines exchange rate modeling for two cases: (a) when the trading partners have mutual interests and (b) when the trading partners have antagonistic interests. Exchange rates in world markets are determined by supply and demand for the currency of each state, and states may control the exchange rate of their currency by changing the interest rate, the volume of credit, and product prices in both domestic and export markets.

Abstracting from issues of production and technology in different countries and also ignoring various trade, institutional, and other barriers, we consider in this article only the effect of export and import prices on the exchange rate, following the approach of [1]. Departing from [1], we propose a new criterion of external trade activity: each trading partner earns a profit which is proportional to the volume of benefits enjoyed by the other partner.

We consider a trading cycle that consists of four stages: a) purchase of goods in the domestic market with the object of selling them abroad; b) sale of the goods in foreign markets; c) purchase of goods abroad with the object of selling them in the domestic market; d) sale of the goods domestically.

Let \( P_x \) be the local price of the export good when purchased domestically in the amount \( x \) (expressed in money units); \( P_{1x} \) the price of the exported good in foreign markets; \( \Pi_y \) the price of the imported good sold domestically in the amount \( y \) (expressed in money units); \( \Pi_{1y} \) the price of the imported good in foreign markets; \( \lambda \) the foreign currency exchange rate (e.g., the exchange rate of a US dollar in Ukrainian karbovanets).

The parameters \( P, \Pi \) are the domestic reservation prices; \( P_1, \Pi_1 \) are the foreign reservation prices; \( a, a_1, b, b_1 \) are nonnegative parameters representing the elasticity of demand, i.e., the ability of the market to respond to changes in supply: increasing purchases raises the price of the diminishing volume of goods and conversely increasing sales reduces the price of the goods.

Let us consider the optimization of profit from mutual trade

\[
\Pi(x, y, \lambda) = x \lambda (P_{1x} - P_x) + y (\Pi_y - \lambda \Pi_{1y}),
\]

where \( x_n, y_n \) are respectively the export and import volumes measured in physical units, \( x_n = x/P_x, y_n = y/\Pi_y \).

The objective function (5) is the overall trading profit: sales revenue less the cost of purchasing the goods in domestic and foreign markets.

The problem is

$$\max_{x, y} \Pi(x, y, \lambda),$$

where

$$\Pi(x, y, \lambda) = \frac{x(\lambda P_1 - P_x) + y(\Pi_y - \lambda \Pi_{y1})}{P_x} = \frac{(\lambda P_1 - P)x - (a + a_1)x^2}{P + ax} + \frac{(\Pi - \lambda \Pi_1)y - (b + b_1)y^2}{\Pi - by}. \tag{7}$$

The problem reduces to solving two equations for $$x, y$$:

$$\frac{\partial \Pi(x, y, \lambda)}{\partial x} = \frac{P(\lambda P_1 - P) - 2(a + a_1)P_x - a(a + a_1)x^2}{(P + ax)^2} = 0, \tag{8}$$

$$\frac{\partial \Pi(x, y, \lambda)}{\partial y} = \frac{\Pi(\Pi - \lambda \Pi_1) - 2(b + b_1)\Pi y + b(b + b_1)y^2}{(\Pi - by)^2} = 0.$$

Market equilibrium is understood in the sense of choosing the export and import amounts that maximize the profit (5). Then the optimal export and import functions are solutions of Eqs. (8) given by the expression

$$x_* = x_*(\lambda) = -\frac{P}{a} + \frac{\sqrt{\frac{P^2}{a^2} + \frac{P(\lambda P_1 - P)}{a(a + a_1)}}}{1 + \frac{a + a_1}{a} - 1}, \tag{9}$$

$$y_* = y_*(\lambda) = \frac{\Pi}{b} - \sqrt{\frac{\Pi^2}{b^2} - \frac{\Pi(\Pi - \lambda \Pi_1)}{b(b + b_1)}} = \frac{\Pi}{b} \left(1 - \sqrt{1 - \frac{1 - \lambda P_1/\Pi}{1 + b_1/b}}\right). \tag{10}$$

By (9), (10), the conditions $$x_* \geq 0, y_* \geq 0$$ are equivalent to the constraints

$$\frac{P}{P_1} \leq \lambda \leq \frac{\Pi}{\Pi_1}, \tag{11}$$

i.e., to ensure profitability of export and import the exchange rate of the exported good $$P/P_1$$ based on reservation prices should not exceed the nominal exchange rate $$\lambda$$, and similarly the reservation-price exchange rate of the imported good $$\Pi/\Pi_1$$ should not be less than the nominal exchange rate.

It is easy to see that the radicands in (9), (10) are always nonnegative. The additional root in (9) with the sign — and the root in (10) with the sign + are rejected, because in the first instance we get $$x_* \leq 0$$ and in the second $$y_* \geq \Pi/b$$. In both cases, the trade activity is unprofitable.

Differentiating, we easily see that

$$\frac{\partial^2 \Pi(x, y, \lambda)}{\partial x^2} \leq 0, \quad \frac{\partial^2 \Pi(x, y, \lambda)}{\partial y^2} \leq 0,$$

i.e., the solutions (9), (10) are indeed maximum points.

When choosing the optimization criterion $$\lambda$$, we should take into consideration the fact that among friendly nations this parameter must not be used as a means for enrichment of some countries at the expense of others. As in security treaties that are based on a minimum level of armament, the choice of the exchange rate also must be aimed at profit minimization, not maximization. This will ensure that foreign currency is indeed used for its originally intended aim — a tender for settling of mutual accounts. In this approach, the exchange rate $$\lambda$$ is determined by solving the minmax problem.