Let $F$ be a degenerate Kleinian group with limit continuum $K$. Then the linear combinations of the fractions $\frac{r_j}{T_j}, T \in \Gamma, j = 1, \ldots, n(\Gamma)$, are dense in $C(K)$ and $A^\alpha(K)$. Bibliography: 6 titles.

Let $\Gamma$ be a degenerate Kleinian group (see [1, Chap. 9, G]), i.e., an analytically finite Kleinian group whose unique invariant component $\Omega$ is simply connected. Assume that $\infty \notin \Omega$; then the limit set $K$ of $\Gamma$ is a bounded continuum without interior and is invariant under $\Gamma$. We can define natural classes of functions on $K$: $C(K)$ is the set of continuous functions on $K$; $A^\alpha(K)$ ($0 < \alpha < 1$) is the Hölder class consisting of all functions $f$ satisfying $|f(z) - f(\xi)| = o(|z - \xi|^\alpha)$ as $|z - \xi| \to 0$. The norm in $A^\alpha$ is given by:

$$||f||_{A^\alpha} \triangleq ||f||_{C(K)} + \sup_{z, \xi \in K} \frac{|f(z) - f(\xi)|}{|z - \xi|^\alpha}.$$

Moreover, as in [2] by E. M. Dyn'kin, we can introduce the spaces $A^\alpha_0(K)$ for $0 < m < \alpha < m + 1$. Namely, $A^\alpha(K)$ consists of functions $f$ for which there exist the (suitably defined) derivatives $f'(z), \ldots, f^{(m)}(z), z \in K$, and $f^{(m)} \in A^\alpha_0(K)$.

It is well known that, for a continuum $K$ with empty interior, the rational functions with poles off $K$ are dense in $C(K)$ (I have not come across a similar statement for $A^\alpha(K)$ in the literature). It turns out that, for the above specific continuum $K$, one can even construct rational functions forming a dense subset of $C(K)$ and of $A^\alpha(K)$ and having poles at points of a fixed countable set naturally related to $K$.

**Theorem 1.** Let $D_{\rho_0} = \{\xi \in C \setminus K : \text{dist}(\xi, K) \geq \rho_0\}$. (a) There exists $n = n(\rho_0, \Gamma, K)$ such that, if some points $a_1, \ldots, a_n \in D_{\rho_0}$ are not $\Gamma$-equivalent, then the linear combinations of the functions $\frac{r_j}{T_j}, T \in \Gamma, j = 1, \ldots, n$, are dense in $C(K)$. (b) Under the same assumptions, the linear combinations of the functions

$$\frac{1}{T(\xi) - a_j}, \quad T \in \Gamma, \quad j = 1, \ldots, n,$

are dense in $C(K)$.

**Theorem 2.** Let $\alpha > 0$ be a noninteger. There exists $n = n(\rho_0, \Gamma, K, \alpha)$ such that, if some points $a_1, \ldots, a_n \in D_{\rho_0}$ are not $\Gamma$-equivalent, then the linear combinations of the functions $\frac{1}{T(\xi) - a_j}, T \in \Gamma, j = 1, \ldots, n$, are dense in $A^\alpha(K)$.

Before proving Theorems 1 and 2, we establish another statement, which is probably of independent interest. We recall that, by Mergelyan’s theorem (see [3]), the polynomials in $z$ are dense in $C(K)$. As far as I know, a similar assertion for $A^\alpha(K)$ has not yet occurred in the literature.

**Lemma 1.** Let $K$ be a bounded continuum with empty interior and connected complement, and let $\alpha > 0$ be a noninteger. Then the polynomials in $z$ are dense in $A^\alpha(K)$.

**Proof.** First, we consider the case $0 < \alpha < 1$. For $f \in A^\alpha(K)$, let $\omega_k(f; \delta) \triangleq \sup_z |f(z) - f(\xi)|$ ($z, \xi \in K, |z - \xi| < \delta$) be the modulus of continuity of $f$. We extend $f$ to the entire complex plane $C$ in such a way that $\omega(\delta, \xi) \triangleq \omega_f(f; \delta) \leq c\omega_k(j; \xi)$ (see [4, Chap. 1, §7]). By definition, $\omega(\delta) = o(\delta^\alpha), \delta \to 0$. We put

$$f_\delta(z) = \frac{1}{\pi \delta^2} \int f(\xi) d\sigma_\xi,$$

where $\sigma_\xi$ is the surface measure of $K$ at $\xi$.
Assume that we are given $\varepsilon, 0 < \varepsilon < 1$. Fixing $\delta$ so that 
\[ 2\omega(\delta) \leq \omega(\delta) \leq 2\varepsilon, \]
we obtain for $|z_1 - z_2| \geq \tau \delta$ (since $|f(z) - f_\delta(z)| \leq \omega(\delta)$): 
\[ \frac{|(f_\delta(z_1) - f(z_1)) - (f_\delta(z_2) - f(z_2))|}{|z_1 - z_2|^\alpha} \leq \frac{2\omega(\delta)}{\tau \delta^\alpha} \leq \frac{2\varepsilon}{\tau \delta^\alpha} = \frac{2\varepsilon}{\tau \delta^\alpha} = 2\varepsilon. \quad (1) \]

But if $|z_1 - z_2| < \tau \delta$, then 
\[ \frac{|(f_\delta(z_1) - f(z_1)) - (f_\delta(z_2) - f(z_2))|}{|z_1 - z_2|^\alpha} \leq \frac{|f(z_1) - f(z_2)|}{|z_1 - z_2|^\alpha} + \frac{|f_\delta(z_1) - f_\delta(z_2)|}{|z_1 - z_2|^\alpha} \leq \frac{2\omega |z_1 - z_2|}{|z_1 - z_2|^\alpha} < 2\varepsilon, \quad (2) \]
i.e., $f_\delta \to f$ in $\lambda^\alpha(K)$.

Denoting by $P(K, \alpha)$ the closure of the polynomials in $\lambda^\alpha(K)$, we see, by the above, that it suffices to verify the relation $f_\delta \in P(K, \alpha)$ for all $\delta > 0$. Let $D \supset K$ be a domain bounded by a smooth curve $L$, and let $K_\delta = \{ \xi \in C \setminus K : \text{dist}(\xi, K) < \delta \}$. Applying the Green formula, we obtain 
\[ f_\delta = f_1 + f_2 + f_3 + f_4, \quad \text{where} \quad f_1(z) = \frac{1}{2\pi i} \int_L \frac{f(z)}{\xi - z} d\xi, \]
\[ f_2(z) = \frac{1}{\pi} \int_K \frac{\partial f_\delta}{\partial \xi} d\sigma_{\xi - z}, \quad f_3(z) = -\frac{1}{\pi} \int_{K_\delta} \frac{\partial f_\delta}{\partial \xi} d\sigma_{\xi - z}, \quad f_4(z) = -\frac{1}{\pi} \int_{D \setminus (K_\delta \cup K)} \frac{\partial f_\delta}{\partial \xi} d\sigma_{\xi - z}. \]

Since $\text{mes}_2 K_\eta \to 0$ as $\eta \to 0$ and $\partial f_\delta/\partial \eta$ is bounded, it can easily be seen that 
\[ \|f_\delta\|_{C(K)} \to 0 \quad \text{and} \quad \omega_C(f_\delta; \delta) \leq c\delta \log \frac{1}{\delta}, \]
whence, by analogy with (1) and (2), we derive that $\|f_\delta\|_{\lambda^\alpha(K)} \to 0$. Thus, it suffices to prove that $f_1$, $f_2$, $f_4 \in P(K, \alpha)$. We fix $\eta > 0$ and choose a $C^2$-smooth Jordan curve $l$ lying in $K_\eta$ and containing $K$ strictly inside. We formulate a property of the polynomials that were used in [5] (this property has not been singled out earlier).

**Sublemma 1.1. Let $G$ be the inside of $l$, and let $\nu$ be a function analytic in $G$ and continuous in $\overline{G}$. Then the polynomials $P_N$ defined in [5] have the following properties:** 
\[ |\nu(z) - P_N(z)| \leq c\omega_0 \left( \frac{1}{N} \right), \quad z \in \overline{G}, \quad c_0(\delta) = c_0(\nu; \delta); \quad (3) \]
\[ \omega_0(\nu_N; \delta) \leq c_0(\delta). \quad (4) \]

Here (3) is directly taken from [5], and (4) is proved in a similar way.

Applying Sublemma 1.1 to $\nu = f_1$ and $\nu = f_4$ (for them $c_0(\delta) \leq C\delta$), we find, as in (1) and (2), that $f_1$, $f_4 \in P(K, \alpha)$. It remains to analyze $f_2$.

Let $Q$ be a square containing $K$. We split it into congruent squares $q_j (j = 1, \ldots, M)$ with side $\sigma$, and put 
\[ \mu_j = -\frac{1}{\pi} \int_{q_j} \frac{\partial f_\delta}{\partial \xi} d\sigma_{\xi}, \quad \nu_j = -\frac{1}{\pi} \int_{q_j} (\xi - o_j) \frac{\partial f_\delta}{\partial \xi} d\sigma_{\xi}, \quad j = 1, \ldots, M, \]
where $o_j$ is the center of $q_j$. Observe that 
\[ |\mu_j| \leq c\sigma^2, \quad |\nu_j| \leq c\sigma^3, \quad j = 1, \ldots, M. \quad (5) \]