VOLUMETRIC STUDIES OF SPHEROIDS AND LEFT VENTRICALES

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The area–length and multiple–slices for estimating heart chamber volumes from uniplane and biplane angiograms are analyzed mathematically for the ideal orthogonal projection of any prolate spheroid in any orientation. The resulting exact formulae relate volume estimates to spheroid shape and orientation. A study is made of possible errors and of best and worst orientations. The variation with orientation for spheroids of fixed shape is compared to the variation with orientation of left ventricle casts: there is a correlation between 0.57 and 0.68 for all models except the biplane area–length model (0.10). An orientation “correction factor” based on the spheroid is shown to improve the volume estimates of the casts slightly (7% to 13% except for the biplane area–length model). The biplane multiple–slices model gives the best results.

Keywords — Ventricle, Volume, Geometric.

INTRODUCTION

The prolate spheroid, an ellipsoid obtained by revolving an ellipse about its major axis, has served as a volumetric model for the left ventricle (1), left atrium (5), right atrium (3), and right ventricle (2). While these heart surfaces are progressively less spheroidal in shape, there is sufficient constancy of shape for each chamber to provide a high correlation between calculated spheroid volumes and known volumes when the model is applied to biplane angiograms using the area–length (AL) method of Dodge and co-workers (1).

A more general, multiple–slice (MS) model also has been used. The model assumes elliptical cross-sections with parallel axes which are perpendicular to
some axis of the heart chamber in question. The model includes prolate and oblate spheroids, ellipsoids that are not obtainable by revolution, and many nonellipsoid surfaces. Numerical integration of the cross-section area as a function of position on the axis yields a volume estimate. This method gives similarly high correlations for heart chamber calculations (4).

Both methods overestimate ventricular volume: a ventricle encloses on the average less volume than its related model surfaces. But regression analysis shows that for the left heart (or right), the true ventricular volume $V$ is approximately proportional to the model volume $M$. That is to say, the linear regression equations have the form

$$V = fM + b$$

with $b$ almost zero and $0 < f < 1$. For the MS method this “proportionality” means that the ventricular cross-section areas are on the average some fraction of the corresponding elliptical areas and this fraction approximates the number $f$ for many ventricles.

Although the MS method is applicable in principle to a more inclusive class of surfaces than the AL methods, the two methods are almost equally effective for a heart chamber. We did find, nevertheless, in our clinical studies that the two methods gave very different volume estimates — infrequently, but often enough to warrant explanation. Mathematical investigation revealed that even a simple spheroid can give disparate volumetric values in certain orientations.

Accordingly, we asked the following questions:

1. What are the exact formulae for MS and AL volumes of any spheroid in any orientation?
2. Given that the MS or AL volume varies with orientation for a simple spheroid and also for a left ventricle cast, is there any similarity between the two variations?
3. Is there enough similarity in this variation to enable us to improve ventricular estimates by correcting for orientational variations as if ventricles were spheroids?

In Appendix B we argue that the spheroid can reasonably replace the ellipsoid with no change in area-length calculations of volume — simply a reinterpretation.

I. THE SPHEROID: MATHEMATICAL RESULTS

We make the simplifying assumption that all projections are orthogonal — i.e., the projecting rays for a given view are mutually parallel, and are perpendicular to the plane of projection. This should give a good first approximation