HYPERSONIC GAS FLOWS AROUND TWO-DIMENSIONAL
OGIVAL BODIES IN THE PRESENCE OF FOREIGN PARTICLES

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The motion of solid particles in a hypersonic two-phase flow of a nonviscous compressed layer in a vicinity
of the point of a two-dimensional ogive is investigated. An analytical solution is obtained, which provides
the means, as distinct from a numerical solution, for writing equations for particle trajectories and finding
coefficients of particle sedimentation. Bibliography: 3 titles.

The problem is considered under the assumption that the equation of motion of the dispersed component
may be reduced to the equation of motion of a single particle in a certain velocity field of the gaseous
component. In contrast to numerical solution [1], for analytical solution approximate methods are used
within the framework of the theory of small perturbations.

We shall study a hypersonic two-phase flow of constant density, with due regard for the main characteristic features of the flow [2]. In this flow a thin symmetric two-dimensional ogival body moves (see
Fig. 1). We assume that until the body meets a shock wave it does not affect the particle motion.

With such simplifications, the analytical solution to the equation of particle motion is obtained, which enables one to calculate quite easily the particle trajectories in the interspace between the body and the
shock wave and, also, to find the sedimentation coefficient E, which is of considerable importance in problems
of this kind.

In order to solve the problem, we must have solutions describing the main parameters (density, pressure,
and velocity) of the hypersonic nonviscous flow in the interspace between the body and the shock wave.
These solutions were obtained in [3]. The surfaces of the body and the shock wave were found, which can
be represented in a vicinity of the ogival point as polynomials in x:

\[ y_b = \tau \left( bx + \frac{1}{2} c_1 x^2 + \cdots \right), \]

\[ y_s = \tau \left( x + \frac{1}{2} l c_1 x^2 + \cdots \right). \]

Here \( \tau \) is the dip angle of the shock wave near the point of the body, \( b \) is the ratio of the initial dip angle
of the body to that of the shock wave, \( l \) is the ratio of the radii of curvature; \( c_1 > 0 \) for convex bodies and
\( c_1 < 0 \) for concave bodies.

With the above assumptions, the equation of motion of a single particle is written as the balance of the
inertial and resisting forces acting on the particle. The resistance coefficient is calculated by the formula

\[ C_D = a_i \text{Re}^{-n_i}, \]

which allows for the cross-sectional area of the particle and the Reynolds number \( \text{Re} \) of the particle.

We assume that the resisting force acting on the particle corresponds to one of the following three regimes:

\[ \text{Re} < 1, \quad a_1 = 24, \quad n_1 = 1; \]

\[ 1 < \text{Re} < 10^3, \quad a_2 = 24, \quad n_2 = 3/5; \]

\[ \text{Re} > 10^3, \quad a_3 = 0.44, \quad n_3 = 0. \]
The index $i = 1, 2, 3$ in formula (3) corresponds to small, medium, and large Reynolds numbers; $a_i$ are empirically fitted coefficients of conjugation.

In dimensionless form the main equation of motion may be written as

$$K_i \frac{d\vec{U}}{dt} = \vec{q}^{1-n_i}(\vec{u} - \vec{U}),$$

where $\vec{U} = dx/dt$ is the intrinsic particle velocity, $\vec{u}$ the gas velocity, and $\vec{q}$ the particle velocity relative to the gas.

The relaxation parameter $K_i$ displays the relationship between the particle and gas velocities:

$$K_i = 2^{3+n_i}(3a_i)^{-1}\rho_r\rho^{-1}(rL^{-1})^{1+n_i}\text{Re}_L^{n_i},$$

where $\rho_r$ is the particle density, $\rho$ is the density behind the shock wave, $\text{Re}_L$ is the Reynolds number of the body calculated from the density $\rho$ and viscosity $\mu$ behind the shock wave.

For $K_i \ll 1$ the particles and the gas attain rapidly the same velocity; for $K_i \gg 1$ it takes them a much longer time to attain equal velocities. If $K_i$ is of the order of one, the velocities of the particles are changing constantly in the course of the particle motion. We shall consider the separation-free motion of similar particles with an ideally smooth surface. Neglecting the terms of higher order in (1) and (2), the dip angle of the shock wave near the point of the body may be found from the relation

$$\beta = \tau - \epsilon \Rightarrow \tau = \beta(1 - \epsilon)^{-1},$$

where $\beta$ is the dip angle of the body near the point and $\epsilon = \rho_\infty\rho^{-1}$ is the ratio of densities on the shock wave.

The boundary conditions in front of the shock wave at infinity are such that the gas and particle velocities are equal, so that $U_\infty = u_0 = \cos \beta \simeq 1$ and $V_\infty = v_0 = -\sin \beta \simeq -\beta$. Behind the shock wave the gas velocity parallel to the surface of the body remains unchanged and equal to that of the incident flow:

$$u = 1, \quad v = 0.$$

In the compressed layer the difference $u - U$ is equal to $\epsilon\beta^2$, i.e., is small. Thus, at the cross section of the layer

$$u = U = 1.$$

Since the body does not affect the particles until they meet with the shock wave, the initial conditions for the velocity at time $t = 0$ may be found from the following relation on the shock wave:

$$V_S(x_S, y_S) = 2(K^2 - 1)((\gamma - 1)K^2)^{-1}y'_S(x) = 2(K^2 - 1)((\gamma - 1)K^2)^{-1}(1 + \nu c),$$

where $K = M_\infty$ is a hypersonic similarity parameter, $M_\infty$ is the Mach number of the incident flow, and $(x_S, y_S)$ are dimensionless coordinates of a point $p$ on the shock wave (see Fig. 1).