SYSTEMS ANALYSIS

CONSTRUCTION OF OPTIMAL PROGRAMMED PATHS FOR
THE MOTION OF A ROBOTIC MANIPULATOR*

V. I. Artemenko, E. N. Gordeev, Yu. I. Zhuravlev,
F. M. Kulakov, V. B. Naumov, G. A. Potapchuk,
I. V. Sergienko, Yu. G. Smetanin, R. Urhardt,
A. N. Khodzinskii, and G. Honderd

In this article, we consider the construction of programmed paths satisfying certain optimization criteria for the motion of a robotic manipulator in an environment with obstacles. The problem is solved in two stages. In the first stage, a so-called geometrical path is constructed in the space of generalized (articulated-joint) coordinates. This path is not a function of time: it is a function of some scalar parameter. In the second stage, given the geometrical path, we construct the sought optimal programmed path as a function of time. The construction of the geometrical path includes an analysis of the behavior of the solution on perturbed input data, in particular, on a perturbed description of the environment (the obstacles). This analysis relies on the concept of stability region of the solution, which is quantitatively characterized in our research.

1. INTRODUCTION

The problem of constructing the optimal paths of a robotic manipulator in an environment with obstacles has been studied by many authors, and some interesting and useful solutions have been obtained.

The first significant results were obtained back in the early 1970s. These results deal with the construction of programmed geometrical paths of robots in an environment with obstacles [8, 10] and reduce the problem to nonlinear programming algorithms. The development of these methods has led to the widely popular method of potential functions [9].

The tremendous literature on the construction of robotic geometrical paths can be conventionally divided into two large groups. The first group includes studies that treat the problem as optimization of some functional in the n-dimensional Euclidean vector space subject to constraints on the argument [8-10]. The second group includes studies that treat the problem in a discrete optimization setting [11-16].

Studies that construct programmed optimal paths as a function of time began to be published somewhat later. This was the period characterized by rapid development of optimal control theory, improvement of classical variational methods, work on dynamic programming and optimization methods based on Pontryagin's maximum principle. Construction of optimal robotic programmed paths provided an excellent application of this developing theory. As a result of efforts in this direction, robotics today is endowed with a whole range of methods for the construction of optimal programmed paths [1, 6, 7]. Although in principle these methods may be used in practice, they require tremendous computational resources for software implementation, which is a serious obstacle to their adoption. Further development and improvement of methods for the construction of optimal programmed paths is therefore a highly topical issue.

The present article is a step in this direction.

Optimal programmed paths are constructed in two stages.

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In the first stage, we obtain a so-called optimal geometrical path. The problem is treated as a discrete optimization problem, and is reduced to finding a minimum cost path in a graph from the starting node to the terminal node. This stage produces a discrete sequence \( q^1(l_1), q^2(l_2), \ldots, q^m(l_m) \) of vectors \( q \) of generalized (articulated-joint) coordinates of the robot, which describe the sought programmed geometrical path: here \( l_1, l_2, \ldots, l_m \) are the values of some parameter \( l \), and \( l_{i+1} - l_i = \Delta l = \text{Const} \). Treating this sequence as the points of a continuous curve, we represent the geometrical path by interpolation as a continuous function of the parameter \( l \): \( q = f(l) \).

This function provides the input data for the second stage, which constructs the programmed path as a function of time. This path guides the manipulator along the geometrical path from the initial position to the final position in a minimum time subject to constraints on controls applied by the drives to the articulated-joint coordinates of the robot.

Deviating from standard approaches to the construction of optimal paths, we propose the following technique.

Optimal geometrical paths are constructed using not only methods and algorithms developed for the first stage, but also techniques that investigate the behavior of the solution under perturbations in the input data of the problem [2-5, 11]. The relevance of this approach is not restricted to errors in measurement and calculation of the geometrical characteristics of the obstacle. It is also associated with the basic statement of the problem. After all, instead of complex geometrical objects representing the robot and the real obstacles in the environment, the algorithmic approach uses rough approximations of the real geometrical objects. It is therefore important to be able to estimate the errors introduced by these rough approximations. Error estimation is based on the concept of stability of solution, which is quantitatively characterized in this study.

We also propose a new method for the construction of a time-dependent optimal programmed path that minimizes the time criterion. This method essentially reduces the resource requirements for software implementation.

2. METHOD OF CONSTRUCTION OF OPTIMAL GEOMETRICAL PATHS

2.1. Statement of First-Stage Problem

Consider a manipulator in the form of an open kinematic chain consisting of \( n \) rigid bodies (segments) joined by simple kinematic couples of fifth class (rotational or translational). The first segment is linked by such a couple to the body of the robot, which is stationary in the inertial system of coordinates.

The manipulator configuration in space is characterized by the generalized (articulated-joint) coordinates \( q \). There are \( n \) such coordinates, forming the vector \( q = (q_1, q_2, \ldots, q_n), q \in Q \).

The position of the \( n \)-th manipulator segment (the grip or tool of the robot) is defined by the 6-dimensional vector \( x \), \( x \in X \). Its relationship with the vector of generalized coordinates \( q \) is defined by \( x = f(q) \).

In general, the dimension of the vector \( x \) is less than the dimension of the vector \( q \). Therefore, to each value of \( x \) correspond infinitely many values of \( q \), i.e., we have a case of kinematic redundancy. The geometrical path in the space \( X \) therefore corresponds to infinitely many paths in the space \( Q \), and we have to choose the "best" among these paths in some sense, for instance, a path such that the section between the given beginning point \( q_b \) corresponding to the point \( x_b = f(q_b) \) and the end point \( q_e \) corresponding to the point \( x_e = f(q_e) \) is of minimal length.

In this case, the path of the robotic tool is regarded as a function of some parameter \( x = \varphi(l) \). The initial and the final positions of the tool (\( x_b \) and \( x_e \)) are given. It is required to construct a sequence \( q(l_1), q(l_2), \ldots, q(l_m) \), \( l_{i+1} - l_i = \Delta l = \text{Const} \), such that:

- the initial and the final positions of the tool, \( x_b \) and \( x_e \), satisfy the conditions \( x_b = f(q(l_1)), x_e = f(q(l_m)) \);
- some performance functional \( \Phi \) (to be defined later) is minimized.

2.2. An Approach to the Solution of First-Stage Problem

We introduce some necessary concepts and notation. Partition the space of generalized coordinates \( Q \) into \( m \) nonintersecting parallelepipeds \( Q = \bigcup_{j=1}^m Q_j \), \( Q_j = \{ q^{l_1}_{1}, q^{l_1}_{2}, \ldots, q^{l_1}_{n} \} \), \( q^{l_1}_{k} = \frac{q_{k-1}}{q_{k-1}} \), such that the mapping \( G_{Q_j} \) is defined on the entire set \( Q_j \times X_j \), where \( X_j = \{ f(q) \} \). We say that the code of the configuration \( q \) is \( e_j = (0, 0, \ldots, 0, 1) \).