INTERACTION OF A SHOCK WAVE WITH A CLOUD OF PARTICLES OF FINITE DIMENSIONS

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This article presents results of calculations of the interaction of a shock wave (SW) with a cloud of particles. It is shown that the volume concentration of particles has a substantial effect on the acceleration of the cloud.

1. We will examine a cloud of solid spherical particles in the path of a shock wave. It is necessary to find the parameters of the gas and particles that result from the interaction of the wave with the cloud. The motion of the particles is modeled by a non-collisional kinetic equation, while the motion of the gas is modeled by the averaged equations of a dust-bearing gas. It is assumed that the particles may be dispersed with respect to velocity and size. The given model was described in detail in [1, 2] and can be used in the case when either the particle trajectories do not intersect within the flow region or particle collisions are rare (Kn = d/(6mL) ≥ 1, L being the relative distance travelled by a particle in the cloud, m the volume concentration of particles, and d particle diameter). Ignoring the effects of heat transfer, we write the complete system of equations [1, 2] in the form

\[ \frac{df}{dt} + \nabla \cdot \left( \rho \frac{c_i^f}{\rho} \right) = \rho \left( \frac{p}{\rho} \right)_i, \]

\[ \rho = \frac{\gamma - 1}{\gamma} \rho_1 T_1, \rho_1 = \frac{\gamma_1}{\gamma_2} \rho_2, \rho_2 = \frac{\gamma_2}{\gamma_1} \rho_1, \]

\[ \rho_1 = m_1 + m_2 = \frac{\sigma}{\rho_2} \rho_2^{1/2}, \rho_2 = \frac{\gamma_2}{\gamma_1} \rho_1, \]

\[ \frac{df}{dt} + \nabla \cdot \left( \rho \frac{c_i^f}{\rho} \right) = \rho \left( \frac{p}{\rho} \right)_i, \]

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where summation is performed over repeating indices i; f is the one-particle distribution function in the phase space t, x, v; dV is an element of volume in the velocity space; n is concentration; a is the mean acceleration; p, µ, and c0 are pressure, viscosity, and sonic velocity in the gas; γ is the adiabatic exponent; Re and M are the Reynolds and Mach numbers; m, m, p, p, p are the volume concentrations, mean densities, and true densities of the gas and particles; c is the specific energy of the gas; F is the i-th component of the force of gas-particle interaction, associated with velocity disequilibrium. Here, we calculate the motion of the gas and particles in...
the axisymmetric and plane cases. Since the cloud may move large distance when accelerating, it is best to solve the problem in a system based on the center of mass of the cloud. Directing the z axis along the velocity of the center of mass D and directing r perpendicular to this velocity, we rewrite system (1.1) as follows:

\[
\begin{align*}
\frac{df}{dt} + \frac{f_1}{\partial x} + w_1 \frac{df}{\partial t} + \frac{\partial}{\partial x}(a_1 f) + \frac{\partial}{\partial y}(a_1 f) + k \frac{w_2}{r} f &= 0, \\
\frac{d}{dc} - D, w_1 = w_1', a_1 = a_1' - D, a_1 = a_1',
\end{align*}
\]

Fig. 1

\[
\begin{align*}
\dot{D} &= \frac{dD}{dt}, D = \int \rho_1 f_1 dV dV, \int \rho_1 f_1 dV dV,
\end{align*}
\]

\[
dV = r^2 dr dz, dV = r^2 dr dz, dV = F_1 + F_2 + F_3,
\]

\[
F_1 = \frac{1}{2} \int m(a_1 c - c_1) + a_1' (w_1 - w_1') \int dV dV dV.
\]

Here, k = 0 in the plane case and k = 1 in the axisymmetric case; v_1, v_2, w_1, and w_2 are the z- and r-components of the velocities of the gas and particles; an l superscript denotes the parameters of the gas and particles in the laboratory coordinate system. The remaining notation is the same as in Eqs. (1.1).

System (1.1)-(1.2) is used to calculate the subsonic (M_{12} < 1) and supersonic (M_{12} > 1) motions of the particles in the gas. In the case of supersonic motion, a conical Mach wave forms near each particle. This model does not permit calculation of the shock wave "sitting" on a given particle, but the presence of the wave is accounted for in the relation C_d(M_{12}). Thus, C_d(M_{12} > 1)/C_d(M_{12} < 1) = 2. This makes it possible to correctly describe the averaged supersonic motion of the cloud in the gas. The coefficient \( \xi \) in the formula for \( C_d \) (see Eqs. (1.1)) is chosen on the basis of agreement of the calculated and experimental [3, 4] dependence of the coordinate of a single particle on time x(t) during its acceleration behind the shock front. The line in Fig. 1 shows the results of calculation of the trajectory of a single particle with \( \xi = 0.38 \), while the circles show experimental results for particles of bronze with \( \rho_2 = 8.6 \times 10^3 \) kg/m\(^3\) and d = 180 \( \pm \) 10 \( \mu \)m. The Mach number of the shock wave M_0 = 2.6, and the initial pressure p_0 = 1 atm. It follows from Fig. 1 that the formula chosen for \( C_d \) adequately describes the motion of the single particle within broad ranges of M_{12} and Re.

2. System (1.2) was solved numerically on a computer using an algorithm that consists essentially of the following. A rectangular Eulerian grid is constructed in the plane (r, z). The mesh size of the grid in terms of r and z is 2h, and 2h, respectively. The equations for the gas are written on the Eulerian grid with the use of an explicit finite-difference scheme.