Introduction. The drag-reduction method based on heating of the surface of a flat plate near its leading edge was first suggested by Kazakov et al. in [1]. The total friction drag of the plate was reduced in this case by increasing the stability of gas flow heated near the leading edge and moving afterward over a colder surface. This resulted in a considerable elongation of the laminar flow region in the boundary layer. The laminar-turbulent-transition delay method proposed by Kazakov et al. in [1] was experimentally validated by Belov and Struminskii, et al. in [2, 3].

Nevertheless, for fairly large values of the characteristic Reynolds number, the turbulent part of the boundary layer is much more extended than the laminar part, and further reduction of the viscous drag is possible if one decreases turbulent friction. Thus, the search for methods of reducing the friction drag using various actions on the turbulent boundary layer is of great importance [4].

The turbulent friction on an isothermal surface with temperature $T*_{w}$ higher than the recovery temperature $T*_{~*}$ is known to be smaller than on an adiabatic surface in flow [5, 6]. However, uniform heating of the entire surface involves considerable technical problems, in particular, the necessity of ensuring a reliable heat insulation for a large area to prevent heat losses inside the body. In addition, in this case the energy supplied for heating of the boundary layer exceeds the gain due to the friction-drag reduction.

In the present paper, the friction in a fully developed turbulent boundary layer on a flat plate is studied as a function of energy supply to one or several local regions of the surface, the remaining part of the surface being thermally insulated. In this case, as will be shown below, the integral friction coefficient is smaller by approximately a factor of 2 than in the case of heat energy supplied to the gas in the boundary layer and distributed uniformly over the entire surface.

Statement of the Problem. Let us consider an inviscid heat-conducting gas flow past a flat plate with velocity $u*_{\infty}$, density $\rho*_{\infty}$, and temperature $T*_{\infty}$ at infinity. It is assumed that at a certain distance $l^*$ from the plate leading edge the laminar boundary layer becomes turbulent, and the place where heat is supplied to the boundary layer is located in the developed-turbulent-flow region and has the length $h^*$. The parameters of an undisturbed turbulent boundary layer are completely determined by the Reynolds number $Re_{0} = \rho*_{\infty}u*_{\infty}h*_{0}/\mu*_{\infty}$ ($\mu*_{\infty}$ is the dynamic viscosity in the incoming flow and $h*_{0}$ is the momentum thickness immediately upstream of the heated region) and, as follows from calculations, are independent of the Reynolds number of the transition, beginning with $Re = \rho*_{\infty}u*_{\infty}l*_{1}/\mu*_{\infty}$. For this reason, the choice of the Reynolds number is not essential, because it is responsible only for the position of the beginning of the heating region determined by a specified value $Re_{0}$.

The temperature along the surface of the heating region and behind it markedly changes. Therefore, to describe the turbulent boundary layer, one should use turbulence models that allow a correct consideration of the hereditary effects in this boundary layer which are most clearly manifested in the high-gradient regions.

One should remember that most of the widely used two-parametric and more complicated turbulence models contain empirical constants and functions which were selected by comparing numerical results with experimental data obtained for an incompressible gas flow past a thermally insulated or isothermal surface [7, 8]. Later on, van Driest proposed to extend these models to compressible heat-conducting gas flows based on the assumption that the momentum and energy transfer processes are similar [9]. Only recently have turbulence models been developed which are suitable for an adequate description of the turbulent boundary layer in compressible gas with intense heat transfer over the immersed surface [10]. However, it is not conclusively proved that they can be efficiently used to describe the characteristics of a turbulent boundary layer on an essentially nonisothermal surface, in particular, with a stepwise variation in its temperature and with regions of high longitudinal temperature gradients and other functions of the flow, which was experimentally studied, for instance, by Carvin et al. in [6].

Nevertheless, there are simple algebraic turbulence models that, notwithstanding their locality, simulate fairly well parameters such as the friction coefficient and heat flux even in a nonequilibrium boundary layer with rather high gradients of the parameters along the surface. At the same time, their use requires much smaller computational resources. In particular, the two-layer algebraic model of Cebeci and Smith [11] yields a good agreement with experiments for boundary-layer calculations on an essentially nonisothermal surface [12]. This fact gives us hope that at least qualitatively reliable results can be obtained when this model is used to calculate the flows with heat supply, which are considered in the present paper.

For numerical calculations, it is more convenient to represent the turbulent boundary-layer equations in dimensionless form, which allows one to take into account considerable changes in the boundary-layer thickness caused by surface heating. For this purpose, the local displacement thickness of the boundary layer $\delta^*$ which is a function of the longitudinal coordinate $x$ is chosen as a characteristic vertical coordinate. In this case, the dimensionless coordinate of the external boundary of the computation domain $y_e$ remains constant under any action on the boundary layer, which facilitates substantially numerical calculations.

The dimensionless variables are introduced according to the relations

$$\begin{align*}
x &= \frac{x^*}{l^*}, \quad y = \frac{y^*}{l^*\delta^*(x)}, \quad \delta = \frac{\delta^*}{l^*}, \quad u = \frac{u^*}{u_{\infty}^*}, \\
V &= \frac{v^*}{u_{\infty}^*} - \frac{yu}{\delta} \frac{d\delta}{dx}, \quad \rho = \frac{\rho^*}{\rho_{\infty}^*}, \quad T = \frac{T^*}{T_{\infty}^*}, \quad \mu = \frac{\mu^*}{\mu_{\infty}^*}.
\end{align*}$$

Here $u^*$ and $v^*$ are the longitudinal and transverse velocity components; $T^*$ is the temperature, K; and the $x^*$ coordinate is counted off from the leading edge of the plate.

In this case, the system of equations and boundary conditions for the turbulent compressible boundary layer has the form

$$\begin{align*}
\frac{\partial \rho u}{\partial x} + \frac{\partial \rho u d\delta}{\partial x} + \frac{\partial \rho V}{\partial y} &= 0, \\
\frac{1}{\text{Re}\delta^2} \frac{\partial}{\partial y} \left[ \left( \frac{\mu}{\sigma} + \mu_t \right) \frac{\partial u}{\partial y} \right] &= \rho u \frac{\partial u}{\partial x} + \rho V \frac{\partial u}{\partial x}, \\
\frac{1}{\text{Re}\delta^2} \frac{\partial}{\partial y} \left[ \left( \frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t} \right) \frac{\partial T}{\partial y} \right] &= \rho u \frac{\partial T}{\partial x} + \rho V \frac{\partial T}{\partial y} - (\sigma - 1) M_{\infty}^2 \mu + \mu_t \frac{(\partial u^2)}{\partial y}, \\
y = 0 : & \quad u = 0, \quad V = 0, \quad \frac{\partial T}{\partial y} + (\sigma - 1) M_{\infty}^2 \text{Re}\sigma \delta q_w = 0 \quad (x_0 \leq x \leq x_0 + h), \\
y = y_e : & \quad u = 1, \quad T = 1, \quad \sigma = 0.72, \quad \sigma_t = 0.9, \quad \mu = \frac{T^{3/2}}{T + 114/T_{\infty}^*} \frac{1 + 114/T_{\infty}^*}{T + 114/T_{\infty}^*}, \quad x = 1.4,
\end{align*}$$

where $q_w$ is the heat flux referred to $\rho_{\infty}^* u_{\infty}^*$, $M_{\infty}$ is the Mach number in the incoming flow, $x_0$ is the coordinate of the beginning of the heating region, and $h = h^*/l^*$ is its nondimensional length.

According to the two-layer model of [11], the turbulent viscosity $\mu_t$ in the above variables is described