DESCRIPTION OF SIGNAL FLUCTUATIONS DURING MEASUREMENTS

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We show that a Pearson distribution of the second kind provides a good description of signal scattering in production quality-control, the distribution of radiation intensity in collimated beams, etc. We present formulas for the moments of one-dimensional, two-dimensional, and conditional distributions. We present graphs for densities and distribution functions. An approach to increasing measurement accuracy is described.

In the process of controlling the quality of production, of material and components, measurements of composition and properties of substances in different processes for extraction and refinement, and in many other problems of metrology, it is necessary to describe the distribution of fluctuations in measurement signals that contain components, parameters of quality criteria, radiation intensities in collimated beams, etc.

Signals that contain components, parameters, quality factors and intensities are probabilistic in nature and have an uncountable set of possible values. As a result, they can be correctly described by an appropriate distribution of continuous random variables. The generally required properties of such distributions are the following: continuity, symmetry, unimodality, boundedness of values for random variables in finite intervals, and the possibility of changing the form of the distribution by changing parameters. Such a distribution can also be used to describe large numbers of random variables in various fields of science and technology, so they are worthy of detailed investigation.

Analysis of more than 100 types of distributions of continuous random variables has shown that less than 20 of them are symmetric: the H-distribution, the Captain distribution of the first kind, two-sided exponential distributions, the Laplace distribution, the normal distribution, the arc sine root distribution, uniform distributions, the Pearson distribution of the second kind, the Kramer-Thompson distribution, the arc sine distribution, the uniform distribution, Student's distribution, the logistical distribution, the inverse normal distribution, Tikhonov's distribution, the cosine distribution, Simpson's triangular distribution, etc. Among these distributions, the requirements of boundedness and the possibility of changing the form in an analytically simple way are met most completely only by the Pearson distribution of the second kind (the Pearson family contains 12 kinds of distributions). Some of the features of the Pearson distributions are presented in [1]. The form of the two-dimensional distribution of the second kind is presented in [2]. However, neither the uniform nor two-dimensional Pearson distribution of the second kind have been investigated in detail.

The univariate probability density of a continuous random variable X that is centered with respect to the mathematical expectation \( \bar{x} = 0 \) is of the form [2]:

\[
p(x) = a^{-2\nu} \left( \frac{\Gamma(\nu+1)\Gamma(\nu+1/2)}{\sqrt{\pi}} \right)^{1/2} \left( a^2 - x^2 \right)^{-\nu-1/2} = H_+ (a^2 - x^2)^{-\nu-1/2},
\]

where \( \nu = 1, 2, 3, \ldots, |a| > 0 \), are parameters of the distribution, \( \Gamma(\nu+1) \) and \( \Gamma(\nu+1/2) \) are gamma functions of \( \nu \). Curves of the density distribution \( p(x) \) for a random variable \( X \) according to (1) for various values of \( \nu \) (indicated by numbers on the curves) and values for the half-open interval with \( a = 1 \) are shown in Fig. 1. For the sake of comparison, this figure also shows graphs for the normal distribution of \( n \). It is clear that the curves for the distributions \( p(x) \) have all of the necessary properties, i.e., they are symmetric with respect to mathematical expectation, permit changes in the interval of variation of \( a \),

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Fig. 1. Probability density $p(x)$ for different values of $\nu$ (indicated by numbers on curves) and density of normal distribution (dashed line).

have "tails" with slopes from 80° to 0°, and peak sharply as the parameter $\nu$ changes. With $\nu = 1$, $p(x)$ provides a good description of the flux of radiation in collimated beams from strongly collimated sources and distributions of signals or the contents of components in various materials.