ADDITIVE AND MULTIPLICATIVE STATISTICAL MODELS OF MEASUREMENT SERIES

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A general form of additive statistical models is described as a wide class of stable distributions. Some properties of the corresponding multiplicative models (logstable distributions) are illustrated by means of transformations of moments of the imaginary order. Algorithms for estimating parameters of a subclass of logstable distributions are provided.

Statistical models of a homogeneous series of n independent measurements \( Y_i \) are based on the assumption that in the course of the measuring process stability of all basic influential factors is maintained and the set of secondary factors acts independently of each other and each one only slightly affects \( Y_i \). In this situation it is natural to interpret \( Y_i \) as an independent random variable obeying the same distribution law.

For additive statistical models — where sums of \( n \) independent identically distributed random variables \( Y_i \) are utilized — the probability theory [1, 2] provides a complete solution of the problem for the limiting (as \( n \to \infty \)) case. In this case, after the corresponding centering and normalization (we shall retain the notation \( Y_i \) for these random variables) the sum becomes a member of a wide class of stable distributions (SD) with the shape parameters \( 0 \leq \alpha \leq 2 \) and \(-1 \leq \beta \leq 1\). In Fig. 1, possible types of the density curves of SD are presented [3] which can be symmetric (\( \beta = 0 \)), asymmetric (\( \beta \neq 0 \)), and even one-sided (\( \alpha < 1 \) and \( \beta = \pm 1 \)). The upper bound on the domain of stable distributions corresponds to the Gaussian (or the normal distribution) where \( \alpha = 2 \) and \( \beta \) is arbitrary. The lower bound (as \( \alpha \to 0 \)) corresponds to the degenerate distribution. Strictly stable distributions are the SD for all admissible values of \( \alpha \) and \( \beta \) except for \( \alpha = 1 \) and \( \beta \neq 0 \).

A sum of an arbitrary number of independent \( Y_i \) having the same SD belongs to the same class but with different values of the location and scale parameters. This additive property of SD can conveniently be exemplified by means of the well-known machinery of CF (characteristic functions) of a distribution law. Here CF is the Fourier transform of the probability density

\[
F_{\mu}(\lambda) = \int g(y) e^{i\lambda y} \, dy.
\]

If independent \( Y_1 \) and \( Y_2 \) with the densities \( g_1(y_1) \) and \( g_2(y_2) \) are given, and also \( F_{\mu_1} \) and \( F_{\mu_2} \), then for their sum \( u = Y_1 + Y_2 \) with the density \( h(u) \) the relation

\[
F_{\mu}(\lambda) = F_{\mu_1}(\lambda) F_{\mu_2}(\lambda)
\]

is valid. This relation is easily generalized for an arbitrary number of summands.

It is known [4] that for a strictly stable SD their CF can be written in the form

\[
F_{\mu}(\lambda) = \exp\{-C|\lambda|^\alpha\}
\]

where \( C \) is a complex number.

Utilization of (1) together with (2) shows that the sum of \( n \) independent \( Y_i \) obeying the same SD is also a stable distribution with the same parameter \( \alpha \). For the cases \( \alpha = 1 \) with \( \beta \neq 0 \) the derivation is more complicated but the result is the same (however, the sum \( u \) has an additional shift).
Fig. 1. Possible types of SD density curves.

Fig. 2. Part of the De–Da diagram.