TIME AND FREQUENCY MEASUREMENTS

METHOD OF PROMPT SYNCHRONIZATION OF HIGH-PRECISION TIME AND FREQUENCY STANDARDS ON MOVING OBJECTS

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A new method of high-precision, prompt synchronization of time and frequency standards on board intermediate-altitude spacecraft is described.

The frequencies of high-stability oscillators at widely separated locations are compared today by methods that use measured discrepancies between the time scales [1]. Those methods in essence come down to a comparison of long intervals of time formed by the oscillators, this being done by two measurements of the time scale discrepancies. If \( U_1 \) and \( U_2 \) are the time scale discrepancies at the beginning and end of the interval of observation, respectively, then the desired frequency discrepancy is given by

\[
\Delta f = f_0 \left( \frac{U_2 - U_1}{T_{\text{obs}}} \right) = f_0 \left( \frac{\Delta t}{T_{\text{obs}}} \right),
\]

where \( f_0 \) is the nominal frequency of the oscillators studied; \( T_{\text{obs}} \) is the interval of observation; and \( \Delta t \) is the drift of one time scale relative to the other in the time \( T_{\text{obs}} \) caused by the difference of the real frequencies. Equation (1) is used when stationary oscillators on the ground are compared.

When the frequencies of the oscillators on board spacecraft are compared by this method, allowance must also be made for the first-order and second-order Doppler effects and the gravitational displacement of the frequency. The order in which those effects are taken into account is described in [2].

This method of frequency comparison is the most accurate of all known methods, but it does have considerable disadvantages, i.e., it is not very operative and requires a long measuring (data acquisition) time.

Analysis of Eq. (1) shows that the measuring time is proportional to the root-mean error of determination of the relative frequency difference of the on-board and ground oscillators. If the frequency difference of on-board and ground oscillators must be determined, for example, with a root-mean error \( \sigma_{\text{diff}} = 1 \times 10^{-12} \) for \( \sigma_{\text{at}} \leq 100 \text{ nsec} \), the spacecraft should be observed for \( T_{\text{obs}} = 27 \text{ h} \). This makes the synchronization of the on-board frequency and time measures less operative. The method is not sufficiently accurate in short intervals of time. For example, \( T_{\text{obs}} = 1 \text{ h} \) and \( \sigma_{\text{at}} \leq 100 \text{ nsec} \), the error \( \sigma_{\text{diff}} \) should be of the order of \( 2 \times 10^{-10} \), which does not meet the requirements for present-day space systems [3].

Let us consider a new method of high-accuracy, prompt synchronization of on-board time and frequency standards that can determine the appropriate errors in a limited observation time.

To make the principal conclusions clearer we describe an intermediate-altitude space vehicle, moving in a circular orbit.

1. The Earth is a sphere of radius \( R_E = 6371 \text{ km} \).
2. The satellite moves under the influence of the Earth’s gravity.
3. The Earth’s gravitational pull is always directed toward the center of the Earth and its absolute value \( g \) is determined from Newton’s formula \( g = \mu/R^2 \), where \( \mu \) is a coefficient that is equal to the product of the gravitational constant and the
Fig. 1. Model of motion of an intermediate-altitude spacecraft (OBFTS = on-board frequency and time standards).

Earth’s mass (with sufficient accuracy for solving a number of practical problems we can assume that \( m = 3.986 \times 10^5 \text{ km}^3/\text{sec}^2 \)) and \( R \) is the distance from the center of the Earth to the satellite.

4. The value and direction of the initial velocity (orbital injection velocity) of the satellite is chosen from the condition for obtaining a circular orbit.

With the given assumptions, when the relativistic effects of measurement on a moving object the frequency difference \( \Delta f_{\text{meas}} \) between the frequency \( f_s \) of the oscillator of a stationary object and the frequency \( f_{\text{ob}} \) of the on-board oscillator of the spacecraft is determined by [2, 5]

\[
\Delta f_{\text{meas}} = f_s - f_{\text{ob}} \sqrt{1 - \beta^2 \cos^2 \varphi},
\]

where \( \beta = v/c \) is the relativistic ratio of the velocities of the object \( v \) to the speed of light \( c \), \( \varphi \) is the angle between the radius-vector and the vector of the spacecraft velocity, measured in the reference of the moving object, as shown in Fig. 1.

If Eq. (2) is expanded in a power series in \( \beta \) and terms higher than second order are ignored, we obtain [7]

\[
\Delta f_{\text{meas}} \approx f_s - f_{\text{ob}} (1 - \beta \cos \varphi - 1/2 \beta^2 + \beta^2 \cos^2 \varphi).
\]

Considering only the first two terms of the expansion (3), we obtain an approximate equation for the frequency difference measured on a stationary object

\[
\Delta f_{\text{meas}} \approx f_s - f_{\text{ob}} (1 - \beta \cos \varphi).
\]

We assume that the ground oscillator frequency \( f_s \) is equal to the on-board oscillator frequency \( f_{\text{ob}} \); then Eq. (4) can be written as

\[
\Delta f_{\text{meas}} \approx \beta f_{\text{ob}} \cos \varphi.
\]

When (5) is taken into account, the measured difference of the time scales of the ground and on-board oscillators in the reference frame of the ground oscillator has the form [1]

\[
U_{s-o.b.}(t) = U_0 + \int_{t_{\text{in}}}^{t} f_{\text{ob}} \beta \cos \varphi(t) dt = U_0 + \int_{t_{\text{in}}}^{t} \beta \cos \varphi(t) dt,
\]

where \( U_0 = U_{s-o.b.} \) is the initial value (at the time \( t_{\text{in}} \)) of the measured time scale difference.

The first part of the derivative of the function \( U_{s-o.b.}(t) \) with respect to time is

\[
\frac{dU_{s-o.b.}(t)}{dt} = \beta f_{\text{ob}} \cos \varphi.
\]

The function \( U_{s-o.b.}(t) \) reaches its minimum value (\( \frac{dU_{s-o.b.}(t)}{dt} = 0 \)) at the time \( t_{\text{meas}} \) when \( \cos \varphi(t_{\text{meas}}) = 0 \); then