FUNDAMENTAL PROBLEMS IN METROLOGY

METHOD FOR DETERMINATION OF STRONG INTERACTION CONSTANTS FROM HADRONIC MASS SPECTRA

V. V. Khrushchev

UDC 531.73+53.081

A method is proposed for determination of quark–antiquark strong interaction parameters in the nonperturbative domain of quantum chromodynamics at relatively large distances. The method is based on a relativistic potential model of quasi-independent quarks and allows us, using experimental data on meson mass spectra, to calculate the quark current masses and the string tension constant with high accuracy.

Progress in development of the theory of interactions between particles is associated with explanation of strong interaction effects in the nonperturbative domain of quantum chromodynamics at relatively large interaction distances. Available methods for calculation of the contributions of such effects, starting from basic principles of quantum chromodynamics, encounter great technical difficulties and, despite the tremendous capabilities of modern computer technology, do not allow us, for example, to calculate the mass spectra of bound states of quarks and antiquarks with the required accuracy.

Among the phenomenological models which are used to calculate mass spectra, potential models are the most effective: these were used to accurately calculate the mass spectra of heavy quarkonia for the first time [1-4]. However, on going to light quarks, the calculation accuracy achieved was lost because both relativistic and nonpotential effects had to be taken into account [3]. These effects can be taken into account in relativistic potential models, which should be compatible with the basic principles of quantum chromodynamics and contain a small number of phenomenological parameters.

Let us consider a method for determination of the strong interaction parameters from the meson mass spectra, which after determination of these parameters allows us in turn to calculate the mass spectra of both light and heavy mesons to high accuracy. The method is based on a relativistic potential model of quasi-independent quarks, which at the time led to creation of hadronic “bag” models [5].

In order to determine the quark–antiquark strong interaction constants from the mass spectra of their bound states, we will first solve the inverse problem. Assume that we need to calculate the mass spectra of bound states when the interaction between quarks and antiquarks is specified. Within quasi-independent particle models, this interaction is separated into two parts: the major interaction with some effective mean field, and a rather weak (residual) interaction between particles, which as a rule cannot be taken into account using the total field. Then the mass of the bound system can be rewritten in the form

\[ M = E_0 + E_1 + E_2 + \ldots + E_n + \varepsilon_{\text{res}}. \]  

(1)

where \( E_0 \) is the total field energy; the \( E_i, i = 1, \ldots, n \) are the energies of the constituent particles in the total field; \( \varepsilon_{\text{res}} \) is the residual interaction energy.

Thus the major complication involves determination of the interaction parameters for interaction of each particle with the total field, which makes the major contribution. For simplification of the calculations, the potential of the external field is usually specified in spherically symmetric form.

Within the framework of such a model, let us consider a very simple bound system consisting of two particles: a quark and an antiquark. Then the residual weak interaction at short distances can be included in the total potential and it can be written as the sum of a quasi-Coulomb potential and a confining potential, which describes the interaction in the nonperturbative domain of quantum chromodynamics found at relatively large distances, about 1 fm [6].

Translated from Izmeritel'naya Tekhnika, No. 12, pp. 3-5, December, 1996.
Considering the spherical symmetry of the potential, in the wave functions of the quark and the antiquark, we can separate out the corresponding spherical harmonics and write the basic radial equation of the model in the form

$$\Phi^*(r) + \Phi(r) = \left[ \frac{L^2 + \frac{1}{2}}{r^2} + \frac{\sigma^2 r^2}{4} + i\sigma r - \frac{4\alpha_s}{3r} \left( \nu + \mu_i^2 \right)^2 + c \right] \Phi^i(r).$$  \hspace{1cm} (2)

where the $\Phi^i(r)$ are the radial wave functions of the quark and antiquark; $\mu_i$, $i = 1, 2$ are the quark and antiquark current masses. Let us select the symbols so that $\mu_1 \leq \mu_2$. The quark—antiquark strong interaction constant at short distances $\alpha_s$ in the single-gluon exchange approximation in quantum chromodynamics can be calculated and rewritten as [1-4]:

$$\alpha_s(r) = \frac{2\pi}{11 - 2\ln \left( \frac{\gamma_{qA}}{\gamma_{qA}} \right)} \theta (\lambda, r).$$ \hspace{1cm} (3)

where $N_f$ is the number of quark flavors; $\Lambda_{\text{QCD}}$ is the quantum chromodynamics constant which is connected with the analogous constant $\Lambda_{\text{MS}}$ in momentum space in a modified minimal subtraction scheme [7]:

$$\gamma_{qA} = \gamma_{qA} \exp \left[ \gamma_{qA} - 1 + \left( \frac{11}{6} - \frac{5N_f}{9} \right) \left( 11 - \frac{2N_f}{3} \right)^{-1} \right].$$ \hspace{1cm} (4)

where $\gamma_{qA}$ is Euler's constant. Let us denote by $\lambda$ the relative orbital angular momentum of the quark and antiquark; let us denote as $J$ the total angular momentum of the considered bound state, i.e., the meson, which is the sum of the total spin $S$ and the orbital momentum $L$. The quantity $L_n$ determines the relativistic centrifugal contribution to the potential of the radial equation (2) and is expressed in terms of $L$ and $\alpha_s$:

$$L_n = -\frac{1}{2} \left[ L (L + 1) + \frac{1}{4} \left( 1 - \frac{16\alpha_s}{9} \right) \right]^{1/2}. \hspace{1cm} (5)$$

Thus for a concrete meson with quantum numbers $J^P$ (P is the spatial parity, $C$ is the charge parity), which within the constituent quark model is considered as the $n^JL$ state ($n = n_1 + 1$, $n_1$ is the radial quantum number), the values of $L_n$ will be specified if we make use of the results of quantum chromodynamics (3). The quark and antiquark current masses $m_i$ also can be determined independently of the model under consideration (for example, [8]). The undetermined phenomenological parameters $\lambda$, $\sigma$, and $c(L, J)$ remain, which describe the contributions of the total field, the possible nonpotential effects, and the effects of the nonperturbative interaction between the quark and the antiquark at relatively large distances.

Since quarks may have higher space—time symmetry [9], Eq. (2) can be a phenomenological approximation of the exact equation for quarks as $H \to \infty$.

As we know, the greatest difficulty within phenomenological models such as the one under consideration is making an adequate choice for the phenomenological parameters. Therefore let us introduce the quantum numbers $j_1$ and $j_2$ of the effective angular momentum of the quark and antiquark in the $n^{2s+1}L_j$ state of the meson as follows:

$$i_1 = i_2 = J + 1/2, \hspace{1cm} \text{if} \hspace{1cm} J = L + S;$$

$$i_1 = j_2 = 1 = J + 1/2, \hspace{1cm} \text{if} \hspace{1cm} J = L - S. \hspace{1cm} (6)$$

According to the symbols used, the condition $\mu_1 \leq \mu_2$ is satisfied. The radial quantum numbers of the quark and antiquark in this case should be equal to each other, $n_i = n_1 = n_2 = n - 1$.

Now let us determine the spectral functions or terms for the quark and antiquark in the $n^{2s+1}L_j$ state of the meson. If $\nu$ is the eigenvalue of Eq. (3), then the spectral function $E_i(n_i, j_i)$ of the quark or antiquark is specified by the formula

$$E_i(n_i, j_i) = \left( \nu + \mu_i^2 \right)^{1/2} + c \left[ 1 - (-1)^{i_1 - i_2} \right]. \hspace{1cm} (7)$$

where $c$ is a phenomenological parameter, equal to 0.035 GeV.