DIRECT ALGORITHMS FOR DATA ANALYSIS IN
THE LOCATION OF SOURCES OF RADIATION
FROM MEASURED DIFFERENCES IN
DISTANCE: MODEL EXPERIMENT

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Results are presented from a model (numerical) experiment performed to check the efficiency of three groups
of direct data-analysis algorithms to be used in satellite location of ground or ocean sources of radiation. The
methodology behind the experiment is also explained.

In locating sources of radiation on the basis of measured differences in distance (the difference-range-finding method
used in radio navigation [1, 2] and optical survey-search systems [3]), the measurement data is usually analyzed with the use
of iterative algorithms based on local linearization of the dependence of the sought quantities — the coordinates of the source
— on the measurement results [4]. From a computational standpoint, however, noniterative direct algorithms are more efficient.
In the latter, the dependence is represented by closed analytic expressions [5-9]. The relative computational simplicity of these
algorithms and the reliability of the resulting estimates, as well as the absence of problems related to the choice of initial values
and the convergence of the results, make direct algorithms promising for all cases that require the analysis of a stream of data
in real time with a very low probability of estimation errors.

In this article, we present results of numerical model experiments performed to check the efficiency of algorithms of
the given type for determining the location of isotropic sources of pulsed radiation with the use of a global network satellite
system of the GLONASS or NAVSTAR type [10, 11].

There are three types of direct (noniterative) algorithms for data analysis that can be used for passive satellite location
of radiation sources on the basis of measured differences in the distance $d_{ij}$ from a source with the coordinates $(x, y, z)$ to i-th
and j-th receivers (RCs) located on different spacecraft (SC) of a global satellite observation system with the coordinates $(x_i, \ y_i, \ z_i)$ and $(x_j, \ y_j, \ z_j)$, respectively. Following the terminology in [8], we will refer to these algorithms as the method of
intersecting spheres (MIS), the method of interpolating spheres (MNS), and the method of intersecting planes (MIP). Analytic
expressions have been obtained to estimate the coordinates of the source, and theoretical formulas have been derived to
determine the covariance matrix of the coordinate estimation errors in first-order perturbation theory [12]. The main theoretical
relations are presented below.

The MIP-estimate of the parameters — the coordinates of the source $x = [x, y, z]^T$ — based the least squares (LS)
criterion is given by the expression

$$\hat{x}_{MIP} = M^+ (A - \hat{R}_0 \hat{d})$$

Here, $\hat{R}_0 = \sqrt{x^2}$ is the estimate of the distance $R_0$ from the source to the reference RC; $M^+$ is a rectangular $3 \times (N - 1)$
matrix that is the pseudo-inverse of the matrix of observations $M$, which comprises the coordinates of $N - 1$ points
representing the location of the receiver; $\hat{d}$ and $\Delta$ are data vectors. The estimate is made in a coordinate system whose origin
coincides with the location of the reference RC.
The covariance matrix $\text{cov}\{\hat{x}\} = E\left[\hat{x}\hat{x}'\right]$ of the estimation errors $\hat{x} = x - \hat{x}$, the diagonal elements of which correspond to the variances of the estimates of the source coordinates in accordance with Eq. (1),

$$\text{cov}\{\hat{x}\} = \begin{bmatrix} \sigma_x^2 & 0 & \cdots & 0 \\ 0 & \sigma_y^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_z^2 \end{bmatrix}$$

has the following form in the first approximation of perturbation theory

$$\text{cov}\{\hat{x}\}_{\text{MIS}} = S \text{diag}\left(\left(d_i + R_s\right)^2\right) E S'.$$

(2)

Here, $S = \left[I + M' dx' / R_s\right]^{-1}$, $M' = S_i M'$, $d_{ij}$ is the difference between the distances from the source to the $i$-th RC and the $j$-th RC chosen as the reference receiver; $E$ is the covariance matrix of measurement errors; $I$ is a unitary matrix.

In the special case of uncorrelated measurements of the same type, when $E = \sigma^2 I$, Eq. (2) can be rewritten as

$$\text{cov}\{\hat{x}\}_{\text{MIS}} = \sigma^2 S \text{diag}\left(\left(d_i + R_s\right)^2\right) S'.$$

(3)

If the source is a long distance from all RCs (as is often the case when the Earth's surface is being probed from outer space), then $R_s >> d_{ij}$, and Eq. (3) is reduced to the form

$$\text{cov}\{\hat{x}\}_{\text{MIS}} = \sigma^2 R_s^2 S_i (M' M)^{-1} S_i,$$

(4)

where $S_i = \left[I + M' dx' / R_s\right]^{-1}$

The MNS-estimate of the vector $x$ in the coordinate system connected with the reference RC is obtained by means of the formula

$$\hat{x}_{\text{MNS}} = (M' P_{\perp}^2 M)^{-1} M' P_{\perp}^2 \Delta,$$

which differs from (1) in that it does not contain a quantity $R_s$ that depends on $x$. On the other hand, this expression does contain a new quantity that depends nonlinearly on $d$, specifically:

$$P_{\perp} = I - dd' / (d'd),$$

- the projection matrix (orthogonal projector), comprising projections onto a subspace orthogonal to the vector $d$.

Given the same assumptions that were made in writing Eq. (4), i.e., $S = \sigma^2 I$, $R_s >> d_{ij}$, the covariance of the estimation errors for the method of interpolating spheres is represented by the matrix

$$\text{cov}\{\hat{x}\}_{\text{MNS}} = \sigma^2 R_s^2 (M' P_{\perp}^2 M)^{-1}.$$

(5)

In accordance with the LS method, the MIP-estimate of the vector $x$ is determined immediately in the initial geocentric coordinate system; it is given by the expression

$$\hat{x}_{\text{MIP}} = M' b,$$

where $b$ is the column vector of the observation data, with the elements

$$b_i = a_{i, \nu} = \frac{1}{2} \left( d_i d_{\mu} d_{\nu} + R_{\mu}^2 d_{\mu} + R_{\nu}^2 d_{\nu} + R_{\delta}^2 d_{\delta} \right).$$

(6)