INVESTIGATION OF THE EFFECT OF YIELDING OF A FOUNDATION
ON THE FREQUENCY AND MODE OF NATURAL VIBRATIONS OF
DAMS BY THE FINITE-ELEMENT METHOD

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UDC 627.824:624.131.522.001.5

Section 5 "Hydraulic Structures" of SNiP II-A.12-69 "Construction in Seismic Regions. Design Standards" was approved by resolution No. 21 of Gosstroi USSR of February 7, 1972, and put into effect on October 1, 1972. The new standard stipulates calculation of hydraulic structures for seismic loads by the dynamic (spectral) method, which takes into account the characteristics of the dynamic working of structures and thus reflects more correctly their behavior during earthquakes. Within the framework of this standard method it is possible to achieve a considerable economic saving by taking into account the yielding of a foundation, which greatly reduces the seismic loads at the lower vibration tones. The effect of foundation yielding on the dynamic behavior of dams can be determined most completely by the finite-element method.

The spectral method calls for expanding the external load (seismic action) with respect to the modes of natural vibrations of the structure. For such expansion it is necessary to know the frequency (period) of natural vibrations and the displacements of points of the structure corresponding to a given vibration mode. These parameters are needed for calculating a structure in the case of any form of assignment of the seismic action (seismicity coefficient or analog accelerogram). Their values are decisive when calculating the design dynamic load. Therefore, it is necessary to have at one's disposal modern, effective, and sufficiently accurate methods of determining the frequencies (periods) and modes of natural vibrations. In particular, this is important for dams having zones differing in configuration and properties of the materials.

A special feature of hydraulic structures which distinguishes them from other types of structures is the large supporting area, which governs the increased effect of foundation yielding on their dynamic characteristics. In addition, these characteristics will be affected also by inhomogeneity of the foundation which is inevitable in view of the considerable length of structures of this type. For example, for the 300-m-high dam of the Nurek hydro development the width at the base exceeds 1.5 km. It is natural to expect that with such ratios of width to height of the dam the effect of the foundation should be considerable.

So far the modes and frequencies of natural vibrations of hydraulic structures have been determined most often according to simplified calculation models in which the structure was divided heightwise into a series of concentrated masses arranged along a vertical line, each of which had only one degree of freedom—in a direction perpendicular to this line. Here it was usually assumed that during vibrations all points of the dam located at the same level are displaced by the same amount, equal to the deflection of a cantilever, which ignored the difference of displacements of individual points over the width of the structure. In such calculation models the foundation was assumed perfectly rigid, and in individual cases its yielding was taken into account by arbitrary methods. For instance, in a number of cases the effect of foundation yielding was taken into account by using the results of studying the vibrations of a rigid plate on an elastic foundation, employing the indirect characteristics of the foundation material (coefficient of uniform and nonuniform elastic compression, modulus of subgrade reaction). Use of the data from experiments performed for quite small plates in comparison to foundations of hydraulic structures frequently resulted in underevaluation of the effect of foundation yielding on the modes and frequencies of natural vibrations of dams and particularly in that for concrete gravity dams on rock foundations the effect of foundation yielding was not taken into account in the calculations.

It is obvious that this approach could not give results sufficiently reflecting the real conditions of the behavior of the "dam-foundation" system under dynamic loads. To determine the modes and frequencies of natural vibrations...
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Among the new methods employing modern computers for solving the problem stated, the finite-element method is promising. This method is used for solving many engineering problems here and abroad, particularly in problems of continuum and structural mechanics [1-4].

In this article the problem of determining the frequencies and modes of natural vibrations of the interacting "foundation-concrete gravity dam" system is solved by the finite-element method. The calculation model of the dam based on the finite-element method represents a plane system of triangular elements of unit thickness whose masses are considered reduced to their vertices. The elastic members between the individual masses are expressed by relations determining the rigidity matrix of the system.

The presence of a yielding foundation under the dam is taken into account by adding to the dam section a 2B × H rectangular region responding to the actions from the dam (B is the width of the dam base and H is its height). The lateral and lower sides of this region were assumed to be fixed rigidly. Preliminary investigations showed that to solve the problem being considered, the adopted dimensions of the added region of the foundation can be considered sufficient.

The problem of the dynamic behavior of a given region of the continuous medium of the dam and the region of the foundation added to it reduces to the problem of the behavior of a set of individual regions—finite elements, into which the original region is divided and which are connected with one another at individual points (vertices of the triangles), called nodes of the system (Fig. 1).

The free vibrations of the system thus obtained are the object of study. The motion of the system is determined by the horizontal and vertical components of the displacement vector \( \{ \delta \} \). The corresponding system of differential equations in a matrix form is:

\[
[M] \{ \ddot{\delta} \} + [K] \{ \delta \} = 0, \tag{1}
\]

where \([M]\) is the matrix of reduced masses; \([K]\) is the rigidity matrix of the structure; \( \{ \delta \} \), \( \{ \ddot{\delta} \} \) are the vectors, respectively, of nodal displacements and accelerations (the dots denote differentiation with respect to time).

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* See also S. B. Ukhov's article in this issue—Editor.

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Fig. 1. Calculation model of the "dam-foundation" system.

Fig. 2. Effect of foundation yielding on the first mode of natural vibrations of the dam. a) \( E_f/E_d = 1/6 \); b) \( E_f/E_d = 1 \); c) \( E_f/E_d = 100 \).