Nonstationary nonisentropic spatial double waves have been studied in [1-5], where some partial solutions of double wave equations were derived. This paper classifies nonisentropic spatial double waves with an arbitrary equation of state \( \tau = \tau (p, S) \) in the presence of functional arbitrariness in the general solution of Cauchy’s problem, which cannot be reduced to invariant solutions.

Consideration is given to nonstationary, nonisobaric, nonisentropic double waves which are irreducible to invariant solutions for the equations of motion of an ideal gas in the spatial case:

\[
\frac{dV}{dt} + \tau \nabla p = 0, \quad \frac{dr}{dt} - \tau \nabla V = 0, \quad \frac{dS}{dt} = 0.
\]

The equation of state \( \tau = \tau (p, S) \) is assumed to display the characteristics \( \tau_p \neq 0 \) and \( \tau_S \neq 0 \). In this case \( V = (u_1, u_2, u_3) \) is the velocity; \( p \) is the pressure; \( S \) is the entropy; \( \tau \) is the specific volume; \( d/dt = \partial/\partial t + u_\alpha \partial/\partial x_\alpha \) (summation over the recurring Greek index is performed from 1 to 3, unless otherwise stipulated).

If \( p \) and \( S \) are functionally dependent on the solution of system (1), then \( p = p(S) \), \( S = S(S) \), and system (1) can be written in the form

\[
\frac{dV}{dt} + \nabla \phi = 0, \quad \frac{d\phi}{dt} = 0, \quad \nabla \nabla V = 0,
\]

where \( \phi = \phi(S) \) are found from \( \phi'(S) = \tau(S) \phi'(S) \neq 0 \).

As double wave parameters we choose \( \phi \) and some function \( \lambda \) which is functionally independent of the wave. From the first two equations of system (2) we obtain

\[
\phi_{ \lambda_i } \partial u_j/\partial \lambda - \phi_{ \lambda_j } \partial u_i/\partial \lambda = 0 \quad (i, j = 1, 2, 3, \ i \neq j).
\]

Continuing (3) \( D/Dt \quad (D/Dt = D_t + u_\alpha D_\alpha) \) and substituting the derivatives \( D\phi_{ \xi_i } /Dt = -\phi_{ \xi_a } ((\partial u_\alpha/\partial \lambda)(\partial \lambda/\partial \xi_i) + (\partial u_\alpha/\partial \phi)(\partial \phi/\partial \xi_i)), \phi_{ \xi_i } = -(\partial u_i/\partial \lambda)(d\lambda/dt) \) we have

\[
\frac{\partial u_i}{\partial \lambda} \frac{\partial \lambda}{\partial \xi_j} - \frac{\partial u_j}{\partial \lambda} \frac{\partial \lambda}{\partial \xi_i} + \frac{d\lambda}{dt} \left( \frac{\partial^2 u_i}{\partial \xi_j} \frac{\partial u_i}{\partial \lambda} - \frac{\partial^2 u_j}{\partial \xi_j} \frac{\partial u_j}{\partial \lambda} \right) = 0 \quad (i, j = 1, 2, 3, \ i \neq j).
\]

As follows from the prohibition of reduction to invariant solutions [6], the rank of the matrix composed of the coefficients for the derivatives \( d\lambda/dt, \partial \lambda/\partial \xi_i (i, j = 1, 2, 3, \ i \neq j) \) in Eqs. (4) and the fifth equation of system (2) must be less than or equal to 2. Hence we arrive at the equation \( \partial u_i/\partial \lambda = 0 \ (i = 1, 2, 3) \) which contradicts the nonisentropicity of the flow. Thus, flows should be considered for which \( p \) and \( S \) are functionally independent.

We choose pressure and entropy as double wave parameters, i.e., assume that \( u_i = u_i(p, S) \ (i = 1, 2, 3) \). Introducing the new dependent variable \( \phi = (\nabla V)/\tau_p \), we reduce system (1) to the form \( (H = \tau_p + u_\alpha u_\alpha) \):

\[
\frac{dp}{dt} - \tau \phi = 0, \quad S_0 = \frac{dS}{dt} = 0, \quad R_1 = u_\alpha S_\xi_a - H \phi = 0, \quad \Phi_i = p_{\xi_i} + u_i \phi = 0 \quad (i = 1, 2, 3).
\]
Differentiating $D_i$ completely with respect to the spatial variable $x_i$ and deriving the combinations below, from Eqs. (5), we obtain

$$D_i \phi_j - D_j \phi_i = u_{ijp} \varphi_{x_i} - u_{ijp} \varphi_{x_j} + (u_{jps} S_{x_i} - u_{ips} S_{x_j}) - \varphi^2 (u_{ijp} u_{ip} - u_{ijp} u_{jp}) = 0;$$  \hfill (6)

$$R_2 = \frac{D (H \varphi - u_{ao} S_{xao})}{Dt} = \frac{H \varphi}{dt} - \varphi (\tau_{u_{ao} S} + u_{ao} u_{bp} u_{bS}) S_{xao} + \varphi^2 (H^2 + \tau H_p) = 0;$$  \hfill (7)

$$D (\phi_i)/Dt = u_{ip} \varphi_{x_i} + \varphi \zeta_{x_i} - \varphi^2 (u_{ip} H - \tau u_{ipp}) = 0 \quad (i, j = 1, 2, 3, \ i \neq j),$$  \hfill (8)

where $\zeta = \tau S + u_{ao} u_{ao}; \ \xi = \tau S + 2 u_{ao} u_{ao}; \ D/Dt = D_t + u_{ao} D$.

Eliminating the derivatives $\varphi_{x_i}$ and $\varphi_{x_j}$ from (6) and using Eqs. (8), we get

$$\left(\zeta u_{ip} - \tau u_{ip} S\right) S_{x_j} - \left(\zeta u_{jp} - \tau u_{jp} S\right) S_{x_i} = 0 \quad (i, j = 1, 2, 3, \ i \neq j).$$  \hfill (9)

Further, as in the case of stationary and plane flows, it is necessary to distinguish between two cases: $H \neq 0$ and $H = 0$.

(1) Let $H \neq 0$. Expressing $\varphi$ in terms of the third equation of system (5) and substituting it into the other equations of the system, together with (9) we derive a homogeneous system of seven quasilinear differential equations with respect to $p$ and $S$. From the prohibition of double wave reduction to invariant solutions [6] it follows that

$$\tau u_{ip} S - \zeta u_{ip} = 0 \quad (i = 1, 2, 3).$$  \hfill (10)

If $u_{ip} = 0 \quad (i = 1, 2, 3)$, then

$$p = h(t), \quad \varphi = h'/r, \quad S_t + u_{ao} (S) S_{xao} = 0,$$  \hfill (11)

where $u_i (S)$ are arbitrary functions; $h(t)$ is a function satisfying the equation

$$h'' = -(h')^2 \partial \ln (|\tau_1|)/\partial p.$$  \hfill (12)

Since $p$ and $S$ are functionally independent, the last equality gives $\partial^2 \ln (|\tau_1|)/\partial p \partial S = 0$ and hence,

$$\tau = A_1(S) g(p) + A_2(S)$$  \hfill (13)

$[A_1(S), A_2(S), \text{and } g(p) \text{ are arbitrary functions}].$ Substituting the latter expression for $\tau$ into (12) and integrating it with respect to the variable $t$, we obtain $g(h(t)) = c_1 t + c_2$ ($c_1$ and $c_2$ are arbitrary constants, $c_1 \neq 0$). Without loss of generality it is assumed that $c_2 = 0$.

Thus, for an equation of state of type (13), double waves exist in which $u_{ip} (S)$ are arbitrary functions of entropy; the pressure $p$ is determined from the equation $g(p) = c_1 t$, and the entropy $S$ satisfies the following system of two differential equations in partial derivatives:

$$dS/dt = 0, \quad u_{ao} S_{xao} = c_1 A_1/(c_1 t A_1 + A_2).$$  \hfill (14)

System (14) is in involution and contains one arbitrary function of two arguments. For instance, for $A_2 = 0$ its solution is

$$tu_1 (S) - x_1 + \psi (tu_2 (S) - x_2, tu_3 (S) - x_3) = 0$$  \hfill (15)

$[\psi (\xi, \zeta) \text{ is an arbitrary function}].$

We now consider the case where $u_{ao} u_{ao} \neq 0$ (for definiteness it is assumed that $u_{1p} \neq 0$). From (10), the existence of the functions $F_i = F_i (p)$ ($i = 0, 2, 3$) follows. Hence,

$$\tau = F_0 u_{1p}^2 - u_{ao} u_{ao}, \quad u_{ip} = F_i u_{1p} \quad (i = 2, 3).$$  \hfill (16)

Excluding the derivatives $d\varphi/dt$ from Eqs. (8) and using Eq. (7) we get

$$\Psi_i = \tau H \varphi_{x_i} + \varphi (H \zeta S_{x_i} + u_{1p} \xi u_{ao} S_{xao}) - \varphi^2 c_i = 0, \quad c_i = 2 u_{1p} H^2 + \tau H u_{1p} - \tau H u_{pp} \quad (i = 1, 2, 3).$$  \hfill (17)

From the combination

$$\tau \varphi \xi u_{ao} D \Sigma R_1 - u_{ao} S (D \Psi /Dt - \tau D \Sigma R_2) + \varphi (\xi u_{ao} u_{ao} + \zeta H) u_{bp} D \Sigma S_0 = 0$$  \hfill (18)