UNEVEN SETTLEMENT OF TALL FOOTINGS
ON NONUNIFORM SOIL

A. V. Vronskii

From the Editors:

In the following article A. V. Vronskii treats the settlement of footings under crane columns and industrial buildings due to nonuniform compressibility of the supporting soils. The author offers a very practical method for computing uneven settlement of tall footings resulting from nonuniform loads for the case of varying yield of the footing base. The same method may be fully applied for the design of long hydraulic structures on layered soils with varying compressibility.

Investigations of existing open crane beam structures at several industrial plants have shown that variable compressibility of soils under footings due to natural soil nonuniformity or inadequate preparation of the footing base leads to considerable tilting causing operational breakdowns [1]. The nonuniformity of the soil at the footing base may even cause uneven settlement of industrial buildings. The above fact points to a need for computing uneven settlement of footings taking into account variable soil compressibility under them.

To describe the variations of soil compressibility, we assume the Winkler hypothesis for foundation behavior, as it is practically the only hypothesis which takes into account variable soil deformation characteristics. The coefficient of subgrade reaction may be found from the soil deformation modulus as the basic foundation compression characteristic by using the familiar ratios between these typical values established by M. I. Gorbunov-Posadov [2]. These ratios were computed assuming that the foundation rotation angles calculated by using the model subgrade reaction coefficient and the elastic half-space are equal. The subgrade reaction coefficient for an eccentric load (the coefficient of nonuniform compression) is computed from the following expression:

$$ c = mc_0, $$

where $m$ is the coefficient found from Table 1 which depends on the footing side ratio $n$ and the direction of the acting moment [2], and $c_0$ is the coefficient of nonuniform compression which is the ratio between the average external pressure at the level of footing to the value of the average settlement of the same footing.

In actual design the average footing settlement may be computed by any standard method, for example, as recommended in the Norms (Design Standards) [3]. Here, the actual footing dimensions, its depth, presence of underground water, etc., are considered automatically. This approach to establishing the footing base compressibility parameter in many cases tends to avoid the shortcomings of the subgrade reaction coefficient, which is an invariable soil characteristic. The degree of soil compressibility under footings (coefficient of subgrade reaction) may be established by computing settlements at the various points of the construction site.

<table>
<thead>
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<th>TABLE 1</th>
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<td>With moment acting along \begin{tabular}{l} the side of footing \end{tabular}</td>
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<td>Longer</td>
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<td>Shorter</td>
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Let us examine the following general case. A tall rectangular footing measuring $a$ and $b$ in plan is loaded by a plane system of vertical forces $P_i$ with eccentricities $e_i$ with respect to the base center at certain heights $h_i$ from the underside of the footing (Fig. 1a).

Assume that the subgrade reaction coefficient is varying linearly as (Fig. 1b):

$$c(x) = c + a\alpha x,$$  \hspace{1cm} (2)

where $c$ is the average value of the coefficient of nonuniform compression within the footing area (kg/cm$^2$), and $\alpha$ is the parameter characterizing the relative nonuniformity of the soil compressibility and is equal to the ratio of the difference between $c_i$ at the different points of the site and the distances between these points (kg/cm$^4$).

Consider the footing to be absolutely rigid. Then the deformation equation may be written

$$w(x) = w_0 + \beta x,$$  \hspace{1cm} (3)

where $w_0$ is the average settlement in cm, and $\beta = \tan \varphi$ is the tilt (see Fig. 1a).

The unknown parameters $w_0$ and $\beta$ are found from the conditions of equilibrium of the underlying soil system: a) the total soil reaction is equal to the external load and b) the sum of the moments of external forces (taking into account the footing rotation) is equal to the moment from the reaction pressure referred to the center of footing. According to the assumed pattern for the footing, the soil reaction is

$$q(x) = c(x)w(x),$$  \hspace{1cm} (4)

and, considering (1) and (2):

$$q(x) = c w_0 + (\alpha w_0 + \epsilon \beta) x + \alpha \beta x^2.$$  \hspace{1cm} (5)

The conditions of equilibrium are

$$\int_{-a/2}^{a/2} bq(x) dx = \sum_{i=1}^{n} P_i,$$  \hspace{1cm} (6a)

and

$$\int_{-a/2}^{a/2} bxq(x) dx = \sum_{i=1}^{n} P_i (e_i + \beta h_i),$$  \hspace{1cm} (6b)

where $b$ is the width of the footing in the plane perpendicular to the exterior load moment.