QUANTITATIVE STUDY OF NONSTATIONARY INTERFERENCE WAVE FIELDS IN LAYERED HOMOGENEOUS ELASTIC MEDIA WITH PLANE-PARALLEL INTERFACES. I. STATEMENTS OF THE PROBLEMS AND EFFICIENT METHODS FOR THEIR SOLUTION

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A new efficient approach to quantitatively studying nonstationary interference wave fields in layered homogeneous isotropic elastic media with plane-parallel interfaces is presented. Adequate statements of problems and efficient methods for their solution are discussed. Quantitative evaluations of fields in different regions of a medium are based on extensive application of contour integration methods, on modern numerical methods for studying the roots of the dispersion equations of problems, and on methods for constructing appropriate stationary contours of phase functions and for computing spectral functions of fields by methods of numerical integration along such contours. Bibliography: 10 titles.

INTRODUCTION

Our decision to focus our attention on a systematic study of interference wave fields in layered media arose from a conviction that the difficulties of modern seismology and seismic prospecting are, in the final analysis, due to the extreme inadequacy of the model concepts of the media under consideration and also to the inadequacy of specific physical consequences of the theory of seismic wave propagation, which could be used in seismologic practice.

1. It is hardly a considerable exaggeration to claim that the above-mentioned physical consequences determining the details of regularities of wave propagation in particular seismic media, on which seismic prospecting can rely at present, belong almost exclusively to the area of describing wave processes within the limits of the zero-order approximation provided by the ray method. However, the ray description of wave processes is essentially high-frequency dominated by its nature. For this reason, for the relatively low-frequency wave fields of seismic practice such a description is partly justified only in the case of large-block, thick-layered models of weakly inhomogeneous seismic media (with smooth interfaces close to mathematical surfaces) such that all their geometric parameters satisfy the inequality \( l \gg \lambda_{\text{dom}} \).

However, such a class of models cannot satisfy the needs of seismic practice because a tremendous number of media encountered everywhere are not covered by it. Among those media are, in particular, different types of heterogeneous formations and thin-layered extended structures generating interference wave fields. Because of this, the need for considerably widening the model concepts in seismology and seismic prospecting (and, at the same time, for devising new methods of theoretical evaluation of wave fields) is now extremely urgent. In this respect, a generalization of methods of the mathematical description of wave fields to the case of seismic media containing, together with thick layers, also thin layers as well as their packets is undoubtedly among the first and foremost problems of wave theory.

2. Developing methods of quantitative treatment of interference wave fields in layered elastic media, we proceed from the simplest class of such media, i.e., from the class of layered-homogeneous isotropic elastic media with plane-parallel interfaces on which the conditions of welded contact are fulfilled (which is assumed, however, only for definiteness). Such a class of models of seismic media provides both a rigorous statement of the problems on propagation of waves excited by arbitrary nonstationary sources and the construction of mathematically exact solutions to them. This yields good reason for regarding the problems of wave propagation in models from the above-indicated class as model problems when studying interference wave fields in general layered media. Moreover, the solution of these problems must provide a foundation for

establishing basic physical regularities of the propagation of nonstationary (interference) wave fields in thin-layered media. Along this line only a few net results have been obtained thus far because of the relative complexity and cumbersoness of the mathematical representations (in the form of iterated integrals) of the solutions to problems of the propagation of interference waves. However, such difficulties proved to be surmountable on the basis of the methods of contour integration and the theory of functions, in close alliance with the algorithmic approaches of numerical mathematics making use of computers. This is precisely what predetermined the publication of this issue containing the first part of our work. Except for general concepts determining our approach to the problem of quantitatively evaluating interference wave fields, it also contains the simplest (but by no means trivial) problems of wave propagation in media comprising only one thin layer and, primarily, a monowave one. For such problems, in the case of a monowave thin layer the roots of the dispersion equations are rather easy to study on the entire complex plane in question. This enables us to present the methods suggested of quantitative treatments of fields in a mostly pictorial way. Consequently, the acquaintance with such problems is a natural springboard for passing to a consideration of interference problems in the case of media of more complicated structure, where one is forced to content oneself with information only sufficient to carry out all the required quantitative evaluations in a justified manner rather than with complete information about the roots of dispersion equations.

In our considerations, elements of methods for treating such problems are illustrated with two simple examples of wave fields of the $SH$ class. The discussion of the results presented provides sufficient evidence for the efficiency of the approaches suggested to the problem. However, a systematic treatment of nonstationary interference wave fields in the case of elastic media comprising packets of thin layers is planned only for our subsequent studies.

Here it remains to point out that (for technical reasons) the present investigation is divided into two parts and is published in two issues of the present series. We include in the first part, presented in this issue, a discussion of questions concerning adequate statements of problems on the propagation of nonstationary interference wave fields and fairly efficient methods of their quantitative treatment. We also present here a number of auxiliary arguments and results primarily related to studying the roots of dispersion equations of the problems under consideration and to realizing numerically the methods of contour integration. The second part of this investigation (to appear in the next issue) will present results of quantitative (numerical) studies of interference wave fields in several particular elastic media comprising thin layers.

3. In concluding this introduction, the authors are pleased to express their gratitude to the Scientific Secretary of POMI, Professor A. P. Oskolkov, for his invaluable support and help in publishing this manuscript. Finally, it should be noted that this publication, split into two parts for technical reasons only, is an outgrowth of the joint efforts of three authors. However, the first part was written by G. I. Petrashen alone and contains only his own results. To the second part (related to applications) all of the authors have contributed evenly, and this is the reason for the triple co-authorship of the whole monograph.

**CHAPTER I. ON NATURAL APPROACHES TO THE QUANTITATIVE TREATMENT OF NONSTATIONARY INTERFERENCE WAVE FIELDS IN LAYERED-HOMOGENEOUS ELASTIC MEDIA WITH PLANE-PARALLEL INTERFACES**

As is well known, the concept of a seismic wave-signal is basic in seismology and seismic prospecting. A wave-signal either carries information about the tectonics of the Earth or sounds the structure of its surface areas. A seismic wave-signal is a nonstationary wave possessing a fairly distinct "datum mark," usually considered to be the shape of a wave or a part of it. Only the availability of such a mark enables one to follow displacements of a wave-signal in the space. That is why this concept is of fundamental importance in the theory of propagation of seismic wave fields.

The sizes of the datum mark of a wave-signal are always comparable with what is meant in seismic practice by the term "the dominating wave length $\lambda_{dom}$ of a signal." The most mathematically precise idea of seismic wave-signals is related to the extremely idealized concept of discontinuous wave fronts. The regularities of their propagation in sufficiently smooth continuously inhomogeneous seismic media are

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1 These second part has been almost completely prepared.