FLUID DYNAMICS OF AN UNDERWATER ELECTRIC DISCHARGE ON THE AXIS OF A SHELL IMMERSED IN A FLUID

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We study the motion of a fluid during an electrical discharge on the axis of an elastic cylindrical shell immersed in the fluid. We estimate the influence of the parameters of the shell on the dynamics of a gas-vapor chamber. We point out several properties of the computation of the pulsed flows in an infinite volume of fluid (the choice of the method of constructing the grid, the limitation of the computational region). We obtain the pressure distribution over space at large distances from the shell. We analyze the influence of the shell on the parameters of the flow in the interior and exterior regions. Four figures. Bibliography: 6 titles

The motion of a fluid during an electric discharge on the axis of a closed cylindrical volume has been studied under various simplifying assumptions: for an incompressible fluid, in acoustic approximation, under constant velocity of expansion of a gas chamber. The author and V. A. Pozdeev [1, 2] have studied the flow of a fluid in gas-dynamic formulation for given laws of expansion of the chamber and in a cylindrical shell. The pulse deformation of cylindrical shells by a fluid during the expansion of a cylindrical piston according to a given law was studied in [3]. The flow was described by the wave equation for the velocity potential, which was integrated numerically. In the present paper we study the motion of a fluid with an immersed shell during an electrical discharge.

Consider a cylindrical shell immersed in water and filled with water. At the initial instant of time an electrical discharge begins on the axis of the shell. The gas-vapor chamber that arises in the discharge causes the fluid to move in the interior region, and that motion is communicated to the surrounding space through the shell. We shall assume that the discharge develops uniformly along the axis of the entire shell, that the gas chamber is cylindrical in shape, that the fluid is ideal and compressible, and that its flow is uniform and isoentropic. Face effects will be ignored. The thin elastic shell is not clamped at the ends, and deforms uniformly over its length. The discharge channel is assumed to have formed at the initial instant of time, and the medium in the discharge channel is assumed to be an ideal gas whose properties do not change. The parameters of the gas are averaged over the whole volume of the chamber and depend only on time. At the initial instant of time the fluid is at rest, and the pressures inside and outside the shell are different.

The flow of the fluid is described by the system of equations of nonstationary gas dynamics, which in cylindrical coordinates can be written as

\[ \frac{\partial p}{\partial t} + \frac{\partial p u r}{\partial r} = 0, \]

\[ \frac{\partial p u r}{\partial t} + \frac{\partial (p u^2 + p) r}{\partial r} = p, \]

\[ p = B \left[ \left( \frac{p}{p_0} \right)^n - 1 \right], \]

where \( B = 304.5 \text{ MPa}, \, p_0 = 10^3 \text{ kg/m}^3, \) and \( n = 7.15 \) are the constants in the equation of state of water in Tait form, \( t \) is time, \( r \) is a coordinate, \( u, p, \) and \( p \) are respectively the velocity, density, and pressure of the fluid.

The system (1) can be solved under the following initial and boundary conditions

\[ u(0, r) = 0, \quad R_{k0} < r < R_c, \quad R_c + \delta_c < r < \infty, \]

\[ p(0, r) = p_{10}, \quad R_{k0} < r < R_{10}. \]

\[ p(0, r) = p_{20}, \quad R_{c0} + \delta_c < r < \infty, \]
\[ u(t, R_k) = u_k, \quad p(t, R_k) = p_k, \quad \text{(3)} \]
\[ u(t, R_c) = u(t, R_c + \delta_c) = u_c. \quad \text{(4)} \]

Here \( R_{k0} \) is the initial radius of the discharge channel, \( R_{c0} \) is the interior radius of the shell, \( \delta_c \) is the thickness of the shell, \( p_{10} \) and \( p_{20} \) are the initial pressure inside and outside the shell, \( R_k \) and \( R_c \) are the current radii of the discharge channel and the shell, \( u_k \) and \( p_k \) are the rates of expansion of the discharge channel and the pressure in it, and \( u_c \) is the velocity of the shell.

The gas parameters in a discharge channel of volume \( V_k \) are connected by the energy balance equation

\[ \frac{dV_k}{dt} + \frac{1}{\gamma - 1} \frac{d(p_k V_k)}{dt} = N_k, \quad \text{(5)} \]

where \( N_k \) is the input electrical power, and \( \gamma \) is the effective adiabatic index of the gas, here assumed to be 1.26.

The dynamic elastic deformation of a thin-walled shell is described by the equation

\[ \frac{d^2 w}{dt^2} + \frac{\sigma_\phi}{\rho_c (w + R_{c0})} = \frac{p_c}{\delta_c \rho_c}, \quad \text{(6)} \]

where

\[ \sigma_\phi = E \ln(1 + w/R_c), \quad \text{(7)} \]

\( w \) is the displacement of the shell, \( \rho_c \) is the density of the material, \( p_c \) is the difference between interior and exterior pressure on the shell, \( \sigma_\phi \) is the principal circumferential stress, and \( E \) is the Young's modulus. Under small deformations \((w \ll R_c)\) one can use the following approximate formula for the stresses:

\[ \sigma_\phi = E w/R_c. \quad \text{(8)} \]

Equations (5) and (6) can be solved with the initial conditions

\[ p_k(0) = p_{k0}, \quad V_k(0) = V_{k0}, \quad \text{(9)} \]
\[ w(0) = w_0, \quad u_c(0) = 0, \quad \text{(10)} \]

where \( p_{k0} \) and \( V_{k0} \) are the initial pressure and volume of the medium in the discharge channel, and \( w_0 \) is the initial deformation, which was determined from Eq. (6) under stationary load.

To estimate the accuracy of the results of computations and the choice of an optimal integration step Eq. (6) was solved numerically taking account of relations (7) and (8) with the pressure \( p_c \) on the shell decreasing exponentially. As computational results showed, for rubber shells under maximal deformation less than 0.1 \( R_{c0} \), the discrepancy between the approximate and exact solutions did not exceed 1%. For that reason we used the simpler relation (8) in subsequent computations.

The problem was solved by the finite-difference methods of Godunov and Wilkins with an artificial viscosity and by the method of characteristics with fixed time step or modifications of it [4]. Test computations showed good agreement of the results obtained by the different methods. However, computations on a computer by the method of characteristics and its modifications require one order less of time than computations by the other methods. For that reason, in studying processes of longer duration (10-20 msec) we used the method of characteristics.

In the computations we employed a moving grid consisting of two regions—interior and exterior. The interior grid was bounded by the surfaces of the discharge channel and the shell, whose laws of motion were unknown in advance and were determined during the computation; the grid step was increased in arithmetic proportion at larger distances from the axis. The exterior grid was bounded by the surface of the shell on one side and was unbounded on the other; the grid step was increased geometrically with distance from the shell. It was established empirically that the grid step should be 0.1-0.05 times the distance from the