MATHEMATICAL MODELING OF VERTICAL ELECTRICAL SOUNDERING OF QUASILAYERED MEDIA

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The direct solution of the vertical electrical sounding problem is considered for three-dimensional quasilayered media. An efficient fast algorithm is obtained for computing the apparent resistivity curves in the two-dimensional case. A PC implementation of the algorithm is proposed.

1. STATEMENT OF THE PROBLEM

Vertical electrical sounding (VES) is one of the main methods of DC electroprospecting [1-3]. In this method, measurements of the electric field on the earth’s surface as a function of the distance between the electrodes are used to determine the distribution of electrical conductivity in the earth’s interior, and thus obtain data on the interior structure of the planet. The solution method for the inverse VES problem has been developed in fair detail for the one-dimensional case, when the conductivity is only a function of depth [4]. It is relevant to develop a solution method for the inverse problem in application to a more general class of nonhomogeneous media, when conductivity is variable in both vertical and horizontal directions. The development of such a method requires an efficient algorithm for the direct VES problem, which computes the electrical field given the sources and the conductivity distribution.

In this article, we propose and investigate a fast method to solve the direct VES method for a quasilayered medium consisting of variable-thickness homogeneous layers:

\[\sigma(x, y, z) = \begin{cases} 
\sigma_0 = 0 & z < z_0 = 0, \\
\sigma_n & z_{n-1}(x, y) < z < z_n(x, y), \quad n \in [1, N], \\
\sigma_{N+1} = 0 & z > z_N(x, y), \end{cases}\]

and \(z_n \to z^0_n = \text{const as } x^2 + y^2 \to \infty\).

The electrical field \(E(M)\) is expressed in terms of the scalar potential \(U(M)\) in the form

\[E(M) = -\text{grad} U(M).\]

The scalar potential is the solution of the boundary-value problem for the Poisson equation

\[\Delta U = -\frac{I}{\sigma_1} \delta(x-x_s) \delta(y-y_s) \delta(z),\]

\[\frac{\partial U}{\partial z} \bigg|_{z = 0} = 0.\]

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with matching conditions at the discontinuity boundaries of conductivity:

$$[U]_{z_n} = 0; \left[ \sigma \frac{\partial U}{\partial n} \right]_{z_n} = 0; \ n \in [1, N],$$

(4)

where $I$ is the source current, $M_S = (x_s, y_s, z_s = 0)$ are the coordinates of the source on the earth's surface.

2. THE INTEGRAL EQUATION

The solution of the boundary-value problem for the potential (3)-(4) is representable in terms of the simple-layer potentials for all the boundary surfaces $S_n, \ n \in [1, N]$, in the form

$$U(M) = U_0(M) + \sum_{n=1}^{N} \int_{S_n} P_n(M_0) G(M, M_0) dS_{M_0},$$

(5)

where

$$U_0(M) = \frac{I}{2\pi \sigma_1 \sqrt{(x - x_s)^2 + (y - y_s)^2 + z^2}},$$

(6)

$$G(M, M_0) = \frac{1}{4\pi} \left( \frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} + \frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z + z_0)^2}} \right),$$

(7)

respectively are the primary potential and the Green's function for the half-space with the boundary condition $\frac{\partial}{\partial z} = 0$ and for $z = 0$.

The potential represented in the form (5) satisfies the equation and the boundary condition for $z = 0$, and also the continuity conditions on the boundaries $S_n, n \in [1, N]$. The potential density $P_n(M), \ n \in [1, N]$, should be chosen so as to satisfy the conditions

$$\left[ \sigma \frac{\partial U}{\partial n} \right]_{z_n} = 0, \ n \in [1, N]$$

(note that for $z_n = z_N$ we obtain the condition $\frac{\partial U}{\partial n} = 0$).

Substituting (5) in the matching conditions for $\frac{\partial U}{\partial n}$ on the surfaces $S_m$, we obtain the system of integral equations

$$\frac{\sigma_m + \sigma_{m+1}}{2} P_m(M) - (\sigma_{m+1} - \sigma_m) \sum_{n=1}^{N} \int_{S_n} P_n(M_0) \frac{\partial G(M, M_0)}{\partial n} dS_{M_0} = (\sigma_{m+1} - \sigma_m) \frac{\partial U_0}{\partial n}.$$

(8)

To normalize, we introduce the functions

$$g(M, M_0) = 4\pi G(M, M_0); \mu_m(M) = \frac{\sigma_1}{I} P_m(M); \kappa_m = \frac{2(\sigma_{m+1} - \sigma_m)}{(\sigma_{m+1} + \sigma_m)}.$$

(9)