MATHEMATICAL MODELS OF PLASMA PHYSICS AND GAS DYNAMICS

APPLICATION OF THE PARTICLE METHOD TO SIMULATE ONE-DIMENSIONAL BOUNDED PLASMA WITH A DISTRIBUTED SOURCES

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We consider the behavior of a plasma bounded in the longitudinal direction by absorbing walls. The model contains charged particles (electrons and ions) moving in the direction of an external magnetic field with two velocity components: longitudinal and transverse. The charged particles are created in pairs by a distributed source. The working model is based on the electrostatic "particles in a cell" method augmented by Emmert's model for a volume source and a model of binary Coulomb particle collisions using the Monte Carlo method. Calculation results are reported for a model with electron-ion collisions and for a collisionless plasma model.

Processes in a finite collisional plasma bounded by an absorbing surface play an important role in applications and in experimental studies. The plasma particles reaching the confining surface are absorbed in it, thus adding to its charge. The electron flux through any cross section in the plasma is greater than the ion flux, so that the condition of approximately equal flux of ions and electrons from a given plasma volume (the condition of stationary state of the system) endows the plasma with a positive potential relative to the walls. In this article, we consider the model of a one-dimensional plasma bounded in the longitudinal direction by absorbing walls. The expected potential distribution is the following: the potential decreases very slowly (in the direction to the walls) in the plasma region where the ion and electron densities are approximately equal (quasineutral plasma), and at the boundary there is a layer several Debye screening length thick where ions predominate. The electric field is essentially nonzero only in this region near the wall.

Theoretical studies of systems considered in this article are based on simplified one-dimensional models [1-4]. All these models assume that the electron density is related with the electrical potential by the Boltzmann equation. Only the motion of particles in the electric field is considered, as the introduction of the magnetic field in these models involves considerable difficulties. A more detailed analysis of these systems should be based on simulation by the particle method. The complete simulation method used in [5-7] takes into account a wide range of effects that are normally ignored in the simplified models. In particular, this approach ensure easy introduction of the magnetic field into the analysis.

The purpose of our study is to develop a numerical code based on the "particles in a cell" method. Unlike the authors of [7], where the field part is based on the solution of the Poisson equation for the electrostatic potential, we determine the electric field from a first-order differential equation.

1. STATEMENT OF THE PROBLEM

We consider the model of a one-dimensional plasma with a three-dimensional source of charged particles and Coulomb collisions between particles. The model is similar to that considered in [7]. A completely ionized plasma is immersed in an external magnetic (oriented in the direction of the z-axis) and is bounded in the longitudinal direction by absorbing walls at points \(z = -L\) and \(z = L\). The entire system is symmetric about \(z = 0\), and it is therefore sufficient to consider only the right half-plane \((0 \leq z \leq L)\).

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The model contains charged particles (electrons and ions) that move in the longitudinal direction (the z-axis) under
the action of forces produced by the own electric field and the external (given) magnetic field. In this model, the particles
have two velocity components: longitudinal ($v_\parallel$) and transverse ($v_\perp$). If the particle crosses the surface $z = 0$, it is reflected
($z \rightarrow -z$, $v_\parallel \rightarrow -v_\parallel$). Particles reaching the collector (the wall at $z = L$) are absorbed. The charge of these particles is added
to the collector charge, which is maintained under a so-called “floating potential.” Initially at $t = 0$ the entire interval ($0 \leq z \leq L$) is uniformly filled with charged particles (electrons and ions) so that the volume charge at each point of the interval
is zero. The particle velocities are chosen from the Maxwell distribution with the temperature $T_0$. The charged particles are
created in pairs (electron–ion) in the volume source region ($0 \leq z \leq L_s$). Moving through the plasma, the charged particles
enter into Coulomb collisions with other particles. The collisions are considered on the entire interval, with the exception of
a thin region near the absorbing wall (the wall layer is a collisionless region).

The motion of electrons and ions in their self-consistent electric field ($E$) and the external magnetic field ($B_\perp$) is
described by the equations of motion in the guiding center approximation:

$$
\frac{dz_s}{dr} = v_{s\parallel}, \quad s = e, i.
$$

(1)

$$
\frac{dv_{s\parallel}}{dr} = \frac{q_s}{m_s c^2} E - \frac{\mu_s}{m_s} \frac{v_\parallel}{v_B} \left( \frac{dv_{s\perp}}{dr} \right)_\text{coll}
$$

(2)

$$
\frac{d\mu_s}{dr} = \left( \frac{d\mu_s}{dr} \right)_\text{coll}.
$$

(3)

Here $\tau = tc$, $c$ is the velocity of light, and the subscript $s$ represents the corresponding particle species ($e$ for electrons and
$i$ for ions); $z$, $q_s$, $m_s$ are respectively the position, the charge, and the mass of the particle; $v_\parallel$, $v_\perp$ are the longitudinal
and transverse velocity components (in units of the velocity of light $c$); $\mu_s = m_s v_\perp^2/2B_0$ ($c^2 m_s$ is the magnetic momentum).

The terms (...)_{coll} in Eqs. (2), (3) describe particle collisions.

The creation of new pairs of particles (electron–ion), e.g., due to ionization by external radiation, occurs in the
volume source $0 \leq z \leq L_s$. The source function $S$ of “injected” particles is described by Emmett’s model [1] in the form
proposed in [7] (except for the absolute value sign):

$$
S_s(z, v_\parallel) = R_{in} h(z) (m_s c/ k T_{d50}) |v_\parallel| \exp(-m_s c^2 v_\parallel^2 /2 T_{d50}).
$$

(4)

Here the subscript $d$ corresponds to the velocity components ($\parallel$ or $\perp$), $R_{in}$ is the number of particles injected in unit volume
in unit time, $h(z)$ is the normalized injection profile, i.e., $\int_0^{L_s} h(x) \, dx = 1$.

The normalizing multiplier in (4) is obtained by integration over the positive velocity half-space. The analysis is
conducted in a cylindrical velocity space. The use of the velocity of “injected” particles from the entire interval as the
longitudinal velocity component ($-\infty < v_\parallel < \infty$) produces an additional multiplier $1/2$ in the right-hand side of formula (4).
This corresponds to the source function from [1], which considers one-dimensional collisionless plasma. Thus, in our study,
particles are “injected” only with positive values of the longitudinal velocity component, which corresponds to using only the
positive part of the Emmett source for this component.

The electrical field $E = -d\varphi/dz$ ($\varphi(z)$ is the electrostatic potential) is the self-consistent field of charged particles. It
is defined by the volume charge density $\rho(z) = \rho_e(z) + \rho_i(z)$. Since the problem is symmetric about $z = 0$, the field $E$ is set
equal to zero. At the boundary $z = L$ we impose the “floating potential” condition $dE/dz|_L = -E(L) = 4\pi \sigma_L$ ($\sigma_L$ is the
charge density on the wall). Under these conditions, the potential $\varphi$ is described by the Neumann boundary-value problem for
the Poisson equation. The solvability condition for this problem in our case is (the Gauss theorem).