NONSTATIONARY INTERACTION OF SUPersonic INVISCID GAS FLOWS

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We consider the flow of supersonic homogeneous gas past a supersonic spherical source. This problem provides a gas-dynamic model of the interaction of interstellar wind with solar wind, and is thus also of independent interest. It is solved using an explicit through divergence scheme of third-order approximation. The analysis focuses on formation and stability of the structure of discontinuity surfaces and convergence to the stationary solution. The results are compared qualitatively and quantitatively with solutions obtained by other methods.

The study considers numerical solution of the problem of interaction of two supersonic ideal gas flows: a flow from a spherical source and a homogeneous supersonic flow impinging on the source. This is a two-dimensional axisymmetric problem. It may be regarded as a gas-dynamic model (ignoring magnetic fields and ionization) of the interaction of the solar wind with interstellar gas. The pure gas-dynamic formulation is also of independent interest. The analysis mainly focuses on the general structure of the solution, its formation and stability. We examine the discontinuity surfaces, the convergence to the stationary solution, and dependence on the parameters of the interacting flows.

This problem has been previously solved by numerical methods in [1, 2]. The through difference method has been used in [1], and the Babenko–Rusanov (BR) method with isolation of discontinuity surfaces has been used in [2]. The structure of the solution known from experiments and calculations is presented in Fig. 1, where the main discontinuity surfaces are shown.

In the nose part, i.e., upstream from the source, these are the two shock waves I and II and the contact surface III between the shocks. In the tail part (downstream from the source), the external shock front (II) and the contact surface extend outside the limits of the numerical region, as in a flow past a blunt body. The internal shock (I) is closed on the axis by a Mach disk (IV) and thus encloses a bullet-shaped region. A break is observed at contact points with the Mach disk: a condensation zone gradually forms downstream from the break along the axis; this condensation zone is separated from the neighboring regions by a shock and a contact surface.

This solution structure is formed with appropriate parameters of the interacting flows, independently of the specific initial state. In particular, initially the system may be without an impinging flow or, alternatively, without a source. For $t > 0$ we specify the following boundary conditions that describe both interacting flows: a) in the impinging flow, we specify the velocity vector $\mathbf{V}$ parallel to the symmetry axis, whose components in polar coordinates are

$$ V_\theta = |\mathbf{V}| \sin(\theta), V_r = -|\mathbf{V}| \cos(\theta) $$

the density $\rho$ and the pressure $p$ are also specified; b) on the unit sphere $r = 1$, we specify the values $V_\theta = 0$, $V_r = V_0$, $\rho = \rho_0$, $p = p_0$. These boundary conditions are augmented with symmetry conditions on the axis $\theta = 0$ and with “soft” boundary conditions on the boundary of the numerical region. We use Euler equations in spherical coordinates and ideal-gas state equations with $\gamma = 1.4$.

The parameters of the interacting flows are normalized by $V = |\mathbf{V}|$, $\rho V^2$ respectively for velocity, density, and pressure. In these dimensionless variables, both flows are completely defined by their Mach number. In our calculations, $M = 3$ and $M_0 = \infty$. Also $V_0/V = 1.25$, $\rho_0/\rho = 1.6$. We use an explicit through scheme of third-order accuracy [3]. The through scheme is expedient for this problem, because the BR method, although more accurate due to isolation of discontinu-
Fig. 1

The solution structure in the tail part is formed 3-4 times more slowly than in the nose part ($t = 50-60$), and no full stabilization of the solution is observed here either. This is evident not only from the fluctuations of the Mach disk, but also from the behavior of the entropy and the Bernoulli integral on the axis downstream from the disk. By the end of the procedure, the fluctuations of these variables had not diminished below 8% and 3%, respectively. For example, the deviations $\delta$ for the Bernoulli integral at different times were the following:

<table>
<thead>
<tr>
<th>$t$</th>
<th>56</th>
<th>63</th>
<th>66.4</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>15%</td>
<td>8%</td>
<td>3%</td>
<td>8%</td>
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