A theorem going back to Maurer asserts that a connected linear algebraic group over the field of complex numbers can be rationally parametrized. In its algebraic formulation this result says that if $G$ is a connected linear algebraic group defined over a field $k$, then (under certain conditions on $k$) the field $k(G)$ of rational functions on $G$ that are defined over $k$ is $k$-isomorphic to a subfield of a purely transcendental extension of $k$. Chevalley has recently given a proof of this when $k$ is any field of characteristic zero, and in addition he has shown that if $k$ is also algebraically closed then $k(G)$ is itself a purely transcendental extension of $k$ [2]. The main result of the present paper extends the first of these results to the case where $k$ is an arbitrary perfect field; as to the second, our information is incomplete. Needless to say, our own methods do not depend on Lie algebras, as do the proofs of Chevalley, and are essentially elementary in nature.

Our paper also contains a number of other results on rationality questions, mostly concerning solvable algebraic groups, and assorted counterexamples. Among results of general interest in the theory of algebraic groups we may call attention to our Propositions 2, 3, and 5, which give strong evidence that the study of the type of field extension obtained by adjoining the characteristic roots of a generic element of an algebraic group of matrices to the field of the generic element will provide much information on the structure of the group. Our final section cleans up some material on fields of definition of generalized jacobian varieties of curves, and gives an example of a connected algebraic group whose maximal connected linear algebraic subgroup is not defined over the same field.

The basic references for this paper are [1] and [5], whose terminology will be followed rather closely (that of [5] taking precedence in a few slight conflicts) and whose results will usually be used without explicit reference. For the general notions of algebraic geometry involved, we refer to [10].
1. Generalities.

A linear algebraic group is an algebraic group which is biregularly isomorphic to an algebraic group of matrices. Note that a distinction is made between the two concepts. However, a linear algebraic group that is defined over a field $k$ is biregularly isomorphic to an algebraic group of matrices that is defined over $k$, the isomorphism also being defined over $k$ [5, Th. 12, Cor. 1], so that in the course of a proof we may without further ado replace a linear algebraic group by a specific biregularly isomorphic matrix group.

In the same way we make a distinction between the concepts «rational homomorphism» and «rational representation», the latter being a rational homomorphism into an algebraic group of matrices.

The word «matrix» will always denote a square matrix of some degree, usually unspecified, whose elements lie in the universal domain. When convenient, a matrix will be considered to operate in the usual way on an underlying vector space (over the universal domain) having dimension equal to the degree of the matrix, and the vector space will be identified with an affine space defined over the prime field. A set of matrices $S$ can be reduced to a set of matrices $S'$ of the same degree if there exists a matrix $a$ such that $S' = aSa^{-1}$; if the elements of $a$ are in a field $k$, we say that $S$ can be reduced to $S'$ over $k$. A semisimple matrix is one which can be reduced to a matrix in diagonal form, a unipotent matrix is one all of whose characteristic roots equal 1. An invertible matrix $a$ can be expressed in one and only one way as the product of a semisimple matrix $a_s$ and a unipotent matrix $a_u$ which commute with each other; $a_s$ and $a_u$ are called the semisimple and unipotent parts respectively of $a$. Under a rational representation of an algebraic group of matrices, semisimple matrices and unipotent matrices are mapped respectively into semisimple and unipotent matrices. Hence it makes sense to speak of semisimple and unipotent elements of a linear algebraic group. The semisimple and unipotent parts of a matrix $a$ are each contained in any algebraic group of matrices containing $a$. Hence if $G$ is a linear algebraic group and $g \in G$, we can write $g = g_s g_u$, where $g_s$, $g_u \in G$ commute and are respectively semisimple and unipotent, and this decomposition is unique; $g_s$ and $g_u$ are called the semisimple and unipotent parts respectively of $g$. Finally, a torus is an algebraic group that is biregularly isomorphic to a direct product $(\mathbb{G}_m)^r$ of multiplicative groups in one variable. An algebraic group of matrices $G$ that is defined over $k$ is a torus if and only if it is connected and can be reduced (over some extension field of $k$) to diagonal form.

For future convenience we bring together in the following lemma and in Prop. 1 a number of easy facts, for the most part well-known.

**Lemma.** Let $g$ be a matrix and let $K$ be a field containing the elements