CALCULATION OF THE LOAD ON A DAVID REDUCTION GEAR WITH OFF-CENTROID EPICYCLOIDAL PINION MESHING

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Planetary reduction gears with off-centroid epicycloidal pinion meshing are used extensively in almost all branches of the engineering industry. The small size and weight of such reduction gears are particularly important for chemical equipment when compared to gears of other types with a similar range of gear ratios. This makes it easy to build a drive into the most diverse equipment.

High gear ratios are obtained by using internal meshing with a difference of one tooth. The technologically most suitable is meshing in which the teeth of the satellite gears are shaped along the equidistant line to a truncated epicycloid and the external gear is made as a pinion.

The most common scheme is K-H-V with symmetrically arranged satellites (Fig. 1a), where an auxiliary mechanism of parallel cranks is used to transform the planetary motion of the satellites into rotational motion of the external member. Most of the reduction gears manufactured by Cyclo (Germany) and Sumitoma (Japan) are based on this scheme. The kinematics and the force analysis of those gears have been the subject of many studies.

The high efficiency of off-centroid pinion meshing in power transmissions with a 2K-H scheme (David scheme, Fig. 1b). Reduction gears of this type are made by Advance Energy (USA). The admission member in them is the carrier H, which communicates the planetary motion to the free satellites 1 and 2. Satellite 1 meshes with the immobile wheel 3, which is mounted on the shaft S. When the numbers of teeth on the gears 0-3 are related by $z_0 = z_1 + 1$, $z_2 = z_1 - 1$, and $z_3 = z_1$, the transmission ratio from the carrier H to the shaft S is $n_{hS} = z_1^2$.

A substantial advantage of reduction gears made according to the 2K-H scheme is that the lack of a parallel-crank mechanism simplifies their production. Prototypes of such reduction gears have been made by the NPO "ÉNIMS" (Scientific-Industrial Firm "Experimental Scientific-Research Institute of Machine Tools").

If planetary-pinion reduction gears based on the 2K-H scheme are to be introduced on a broad scale, the load on the reduction gears must be calculated by a method that takes the distinctive features of the given gears into account. In contrast to reduction gears with the K-H-V scheme, in the David reduction gear the satellites are arranged asymmetrically and, therefore, their radial reactions do not compensate each other. In the ordinary design of the reduction gear the carrier H is connected to the shaft of an electric motor directly by a clutch, i.e., is a high-speed member. In view of this the auxiliary weights $W_1$, $W_2$ (see Fig. 1b) can be used to balance the reactions that are transmitted from the meshing of gears 0-1 and 2-3 as well as the inertial forces.

Cycloidal meshings are more sensitive to a change in the center-to-center distance between gears. An exact picture of the loading, therefore, can be obtained by taking into account both the axial and torsional displacements of the members. The gaps in the kinematic couples must be included, since they usually exceed the elastic contact strains.

We consider the case when the output shaft B is loaded by a constant resisting moment $M_r$. The driving moment of the forces on the carrier H that is necessary for overcoming $M_r$ is denoted $M_d$. The displacement vector $\mathbf{q}$ of the members includes $\alpha_{H0}$ and $\alpha_H$ (the angles of rotation of the carrier H in the sections A-A and B-B), $\alpha_1$, $\alpha_{1x}$, $\alpha_{1y}$, $\alpha_{2x}$ and $\alpha_{2y}$ are the total angles of rotation and the axial displacements of satellites 1 and 2, $\alpha_3$, $\alpha_{3x}$, and $\alpha_{3y}$ are the total angles of rotation of the shaft B and the axial displacements in the section D-D, and $s_{BS1x}$, $s_{BS1y}$, $s_{BS2x}$, and $s_{BS2y}$ are the axial displacements of the shaft S at the bearing seats.

The axial displacements of the carrier H are zero because it is in equilibrium.

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The driving torques of the forces $M_r$ and the disbalances $D_1$ and $D_2$ of the counterweights $W1$ and $W2$ are also unknown.

All of the unknowns mentioned above can be found from the complete system of the equations of kinetostatic equilibrium of the reduction gear members. The total number of unknowns is 19, however, and so solving the complete system is a complicated problem.

The angle of torsion in the section $D-D$ is

$$\alpha_3 = \frac{M_r}{C_s},$$

where $C_s$ is the reduced torsional stiffness of the shaft between the sections $C-C$ and $D-D$.

To obtain a closed system of equations, possibly of lower order, and to simplify the solution of the general problem we express the axial displacement of the gear 3 in terms of the reaction $R_{32}$ in the meshing of gears 2-3:

$$\bar{s}_3 = C_s R_{32},$$

where $C_s = (i_0^2 (l_a + 3l_b)/6E_l) - (i_1^2 (l_a + l_b^2)/J_{BS1}) - (i_2^2 (l_a + l_b)/J_{BS2})$. $J_a$ and $J_b$ are the moments of inertia of sections of length $l_a$ and $l_b$, and $J_{BS1}$ and $J_{BS2}$ are the stiffness coefficients of the bearings $BS1$ and $BS2$.

Substituting the expressions for the projections $s_3x$ and $s_3y$ into the complete system of equations of kinetostatic equilibrium of the reduction gear members, we can separate a closed system of equations in the displacements $s \subset q$, $s = (s_{H1}, s_{1x}, s_{1y}, s_{2x}, s_{2y}, s_{3x}, s_{3y})$

$$R_{H1x} + R_{01x} + 0.5F_{ix} = 0;$$
$$R_{H1y} + R_{01y} + 0.5F_{iy} = 0;$$
$$R_{H2x} + R_{32x} + 0.5F_{ix} = 0;$$
$$R_{H2y} + R_{32y} + 0.5F_{iy} = 0;$$
$$M_{B1} + M_{B2} + M_{01} + M_{32} = 0;$$
$$M_{23} + M_s + M_{B1} + M_{B2} = 0;$$
$$s_{3x} = C_s R_{32x};$$
$$s_{3y} = C_s R_{32y},$$

where $F_{ix}$ is the inertial force of the satellites and $M_{B1}$, $M_{B2}$, $M_{B1}$, and $M_{B2}$ are the moments of the frictional forces in the respective bearings.

Having determined the unknowns in Eqs. (1) and (2), we find the others from

$$M_d = -M_{11} - M_{21} - M_{B1} - M_{B2};$$
$$s_{11} = \frac{M_d}{C_{11}};$$

$$\bar{s}_{BS1} = -\frac{R_{32}}{J_{BS1}} \left( \frac{l_a + l_b}{l_b} \right);$$

$$\bar{s}_{BS2} = -\frac{R_{32}}{J_{BS2}} \left( \frac{l_a}{l_b} \right);$$

$$D_1 = C_D (R_{11}(a + \Delta_2) + R_{21}\Delta_2);$$

$$D_2 = C_D (R_{11}(a \Delta_1) + R_{21}(a \Delta_2));$$

$$C_D = -1/\omega_0^2 (a + \Delta_1 + \Delta_2),$$

where $\Delta_1$, $a$, and $\Delta_2$ are the distances between the central planes of members $W1-I$, $I-2$, and $I-W1$, respectively (Fig. 1b).