Wave Packet Approach to Lattice Electrons in External Galvanomagnetic Fields

III. External Electric Fields

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Received May 1, 1969

The probability for Bragg reflections of wave packets in a lattice and an external electric field \( F \) is no longer invariant against inversion: (a) Reflections anti-parallel to \( F \) are favored compared to reflections parallel to \( F \) owing to the energy gain involved. (b) Reflection amplitudes caused in successive cyclotron periods cancel by interference, except when the respective energy gain equals an integer number of magnetic quanta. The effective current parallel to \( F \) is calculated up to terms of second order in the lattice potential. It is found proportional to \( 1/B^2 \) for scattering times shorter (low field limit) and proportional to \( 1/B \) for scattering times longer (high field limit) than one cyclotron period. The effect of magnetic breakdown interpolates between the results of a band model for closed and open electron orbits. From the reflection condition (b) one can deduct oscillations of the effective current in \( F/\hbar B^2 \) in the \( 1/B \)-region.

A. Introduction

In Part III of this series [1] we are concerned with the effect of an external electric field \( F \). Since the Hamiltonian in an external magnetic field \( B \) contains terms quadratic in \( r \) anyway, we find the electric potential \(-Fr\) not to yield
any analytical difficulty, but merely to create some additional terms. We restrict our investigations to the transverse case $\mathbf{F}$ normal to $\mathbf{B}$, so that the separation of the Hamiltonian from three to two dimensions can be done without regard to $\mathbf{F}$. The main modification of the magnetic wave packets is a factor accounting for the Hall drift (Sects. B, C). The mean electron position observes an equation of motion, which in addition to the quasi-classical Coulomb and Lorentz forces contains contributions from all possible reflections in the orbit lattice (Sect. D). These cause an effective current parallel to $\mathbf{F}$ (Sects. E, F).

### B. Free Electron Orbitals

We assume $\mathbf{F}$ to be parallel to the $x$-direction of our coordinate system. Starting again with the discussion of the electron orbitals for vanishing lattice potential we redo the separation of the Schrödinger equation in the rotating coordinate system $\xi_1(t), \xi_2(t)$ described in Part I, and obtain instead of (1.6)

$$i \frac{d}{dt} a_{1j} = -a_{1j} a_{2j} - F \cos \left( \frac{B}{2} (t - t_0) \right)$$

and by integration of (2) and of the unaltered Eq. (1.11) for $a_{0j}(t)$

$$a_{0j}(t) = [a_{0j}(t)]_{F=0} - i \frac{a_{0j} + a_{2j}}{B \cos \frac{B}{2} (t - t_0)} \left\{ \frac{F t}{2} \sin \left( \frac{B}{2} (t - t_0) \right) + \frac{F \cos \left( \frac{B}{2} (t - t_0) \right)}{2 B \sin \left( \frac{B}{2} (t - t_0) \right)} \right\}$$

where

$$a_{ij}(t) = a_{ij} + \frac{F t}{2} \cos \frac{B}{2} (t_1 - t_0) \pm \frac{F}{2 B} \sin \left( \frac{B}{2} (t - t_0) \right) \left( t - t_0 + \frac{t_1}{2} \right).$$

The most important effect of the electric field $\mathbf{F}$ is the continuous variation of the quasi wave numbers $\xi_j$ with time $t$. It changes the electron orbit from a circle into a cycloid. We denote the new quasi wave numbers in the presence of the electric field by $\xi_j(t)$ and, consequently, the resulting free electron orbitals by $|\xi_1 \xi_2\rangle$. We have

$$|\xi_1 \xi_2\rangle = \exp \Delta_F |\xi_1 \xi_2\rangle$$

with

$$\Delta_F = \sum_{j=1}^{2} \left\{ [a_{1j} - [a_{1j} F=0] \xi_j + [a_{0j} - [a_{0j} F=0]] \right\}.$$